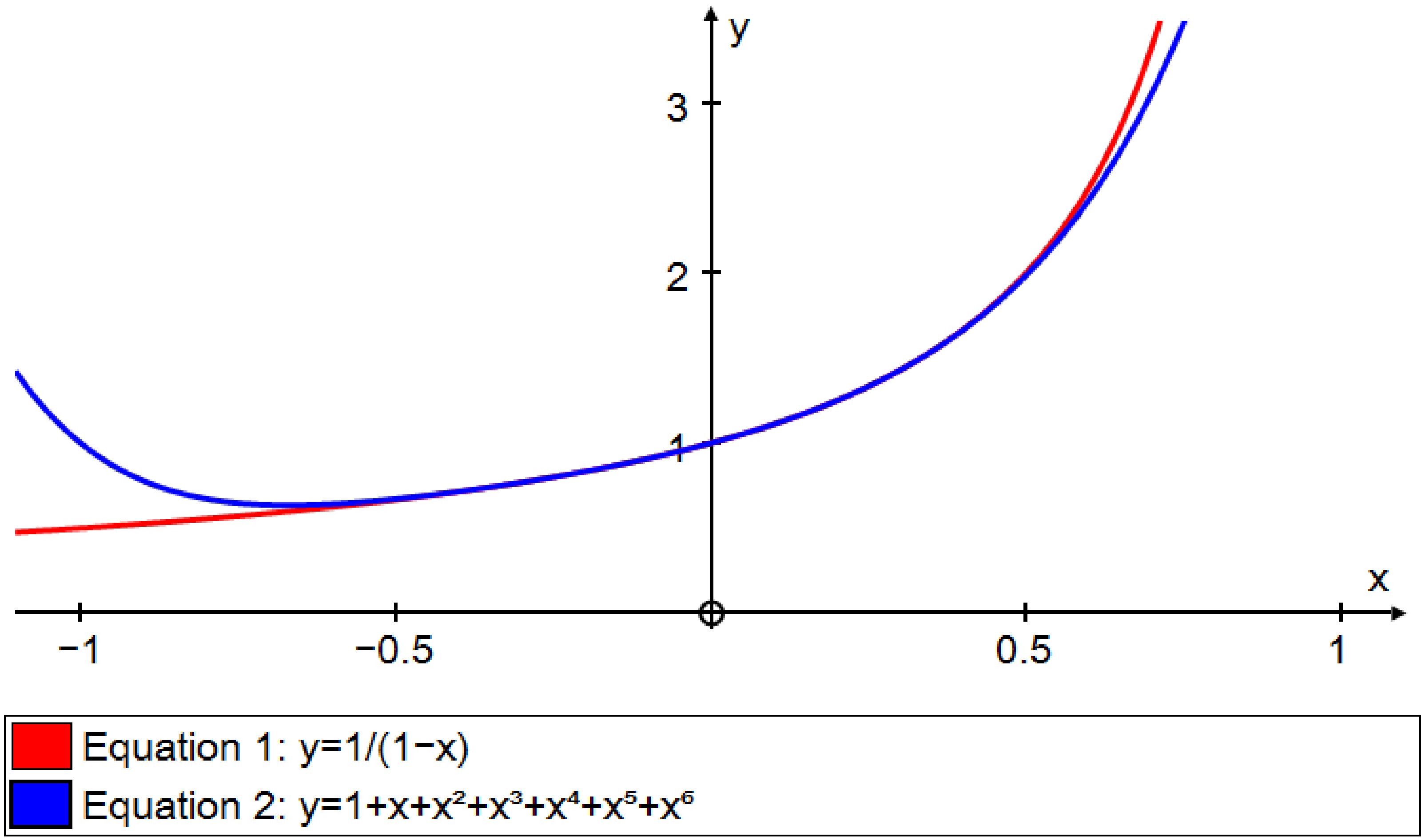
$$\begin{array}{r} 1.\ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 9\ \ 0\ \ 1\ \ 2\ldots \\ 81\overline{)\,100.{}^{19}0\,{}^{28}0\,{}^{37}0\,{}^{46}0\,{}^{55}0\,{}^{64}0\,{}^{73}0\,{}^{1}0\,{}^{10}0\,{}^{19}0\ldots.} \end{array}$$

$$1 + x + x^2 + x^3 + \ldots = \frac{1}{1-x}, \qquad -1 < x < 1$$

$$1+x+x^2+x^3+\ldots=\frac{1}{1-x}, \qquad -1<x<1$$

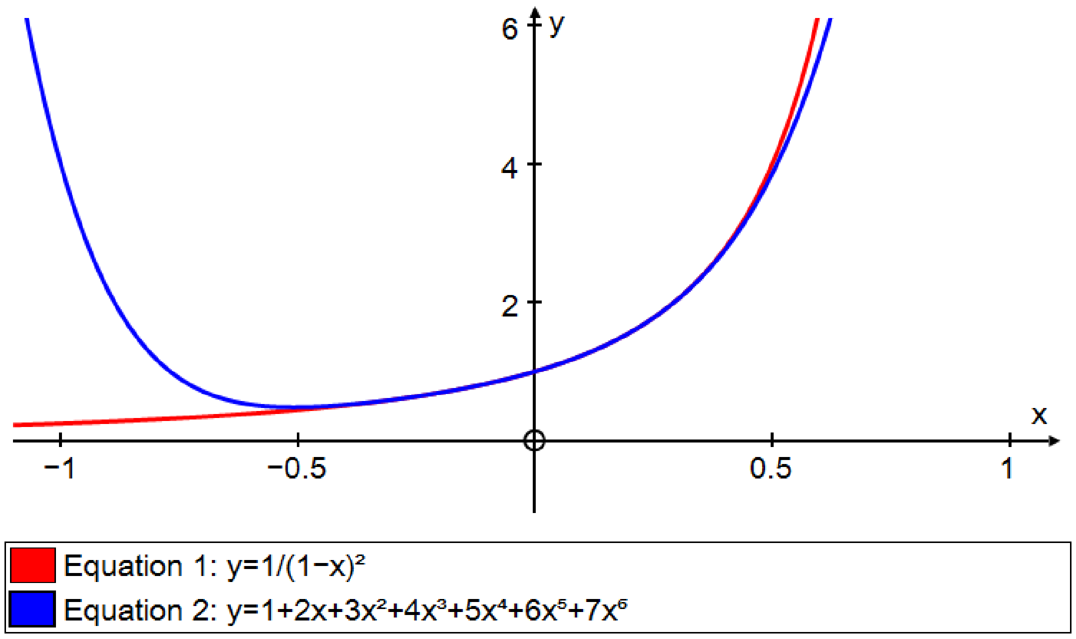


$$1 + x + x^2 + x^3 + \ldots = \frac{1}{1 - x}, \qquad -1 < x < 1$$

$$1 + 2x + 3x^2 + 4x^3 + \ldots = \frac{1}{(1 - x)^2}$$

$$1 + x + x^2 + x^3 + \ldots = \frac{1}{1-x}, \qquad -1 < x < 1$$

$$1 + 2x + 3x^2 + 4x^3 + \ldots = \frac{1}{(1-x)^2}$$



$$1 + x + x^2 + x^3 + \ldots = \frac{1}{1-x}, \qquad -1 < x < 1$$

$$1 + 2x + 3x^2 + 4x^3 + \ldots = \frac{1}{(1-x)^2}$$

$x = 0.1:$

$$1 + 0.2 + 0.03 + 0.004 + \ldots = \frac{1}{0.9^2} = \frac{100}{81}$$

1 .
2
3
4
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7
8
9
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1 1
1 2
1 3
1 4
1 5
1 6
1 7
1 8
1 9

Andrew Palfreyman, in the MA’s Symmetry +

$$\begin{array}{r} 0.\;0\;\;1\;\;1\;\;2\;\;3\;\;5\ldots \\ 89\overline{)\,1.{}^{1}0\,{}^{10}0\,{}^{11}0\,{}^{21}0\,{}^{32}0\,{}^{53}0\ldots} \end{array}$$

$$F_0 = 0, F_1 = 1, \quad F_n + F_{n+1} = F_{n+2}$$

$$R = \frac{F_0}{10^1} + \frac{F_1}{10^2} + \frac{F_2}{10^3} + \frac{F_3}{10^4} + \ldots$$

$$F_0 = 0, F_1 = 1, \quad F_n + F_{n+1} = F_{n+2}$$

$$R = \frac{F_0}{10^1} + \frac{F_1}{10^2} + \frac{F_2}{10^3} + \frac{F_3}{10^4} + \ldots$$

$$R = \frac{\left(F_2 - F_1\right)}{10^1} + \frac{\left(F_3 - F_2\right)}{10^2} + \frac{\left(F_4 - F_3\right)}{10^3} + \frac{\left(F_5 - F_4\right)}{10^4} + \ldots$$

$$F_0 = 0, F_1 = 1, \quad F_n + F_{n+1} = F_{n+2}$$

$$R = \frac{F_0}{10^1} + \frac{F_1}{10^2} + \frac{F_2}{10^3} + \frac{F_3}{10^4} + \ldots$$

$$R = \frac{(F_2 - F_1)}{10^1} + \frac{(F_3 - F_2)}{10^2} + \frac{(F_4 - F_3)}{10^3} + \frac{(F_5 - F_4)}{10^4} + \ldots$$

$$R = \left( \frac{F_2}{10^1} + \frac{F_3}{10^2} + \frac{F_4}{10^3} + \frac{F_5}{10^4} + \ldots \right) - \left( \frac{F_1}{10^1} + \frac{F_2}{10^2} + \frac{F_3}{10^3} + \frac{F_4}{10^4} + \ldots \right)$$

$$F_0 = 0, F_1 = 1, \quad F_n + F_{n+1} = F_{n+2}$$

$$R = \frac{F_0}{10^1} + \frac{F_1}{10^2} + \frac{F_2}{10^3} + \frac{F_3}{10^4} + \ldots$$

$$R = \frac{(F_2 - F_1)}{10^1} + \frac{(F_3 - F_2)}{10^2} + \frac{(F_4 - F_3)}{10^3} + \frac{(F_5 - F_4)}{10^4} + \ldots$$

$$R = \left( \frac{F_2}{10^1} + \frac{F_3}{10^2} + \frac{F_4}{10^3} + \frac{F_5}{10^4} + \ldots \right) - \left( \frac{F_1}{10^1} + \frac{F_2}{10^2} + \frac{F_3}{10^3} + \frac{F_4}{10^4} + \ldots \right)$$

$$R = 100\left( \frac{F_2}{10^3} + \frac{F_3}{10^4} + \frac{F_4}{10^5} + \frac{F_5}{10^6} + \ldots \right) - 10\left( \frac{F_1}{10^2} + \frac{F_2}{10^3} + \frac{F_3}{10^4} + \frac{F_4}{10^5} + \ldots \right)$$

$$F_0 = 0, F_1 = 1, \quad F_n + F_{n+1} = F_{n+2}$$

$$R = \frac{F_0}{10^1} + \frac{F_1}{10^2} + \frac{F_2}{10^3} + \frac{F_3}{10^4} + \dots$$

$$R = \frac{\left(F_2 - F_1\right)}{10^1} + \frac{\left(F_3 - F_2\right)}{10^2} + \frac{\left(F_4 - F_3\right)}{10^3} + \frac{\left(F_5 - F_4\right)}{10^4} + \dots$$

$$R = \left( \frac{F_2}{10^1} + \frac{F_3}{10^2} + \frac{F_4}{10^3} + \frac{F_5}{10^4} + \dots \right) - \left( \frac{F_1}{10^1} + \frac{F_2}{10^2} + \frac{F_3}{10^3} + \frac{F_4}{10^4} + \dots \right)$$

$$R = 100\left( \frac{F_2}{10^3} + \frac{F_3}{10^4} + \frac{F_4}{10^5} + \frac{F_5}{10^6} + \dots \right) - 10\left( \frac{F_1}{10^2} + \frac{F_2}{10^3} + \frac{F_3}{10^4} + \frac{F_4}{10^5} + \dots \right)$$

$$R = 100\left( R - \frac{F_0}{10^1} - \frac{F_1}{10^2} \right) - 10\left( R - \frac{F_0}{10^1} \right)$$

$$F_0 = 0, F_1 = 1, \quad F_n + F_{n+1} = F_{n+2}$$

$$R = \frac{F_0}{10^1} + \frac{F_1}{10^2} + \frac{F_2}{10^3} + \frac{F_3}{10^4} + \ldots$$

$$R = \frac{(F_2 - F_1)}{10^1} + \frac{(F_3 - F_2)}{10^2} + \frac{(F_4 - F_3)}{10^3} + \frac{(F_5 - F_4)}{10^4} + \ldots$$

$$R = \left( \frac{F_2}{10^1} + \frac{F_3}{10^2} + \frac{F_4}{10^3} + \frac{F_5}{10^4} + \ldots \right) - \left( \frac{F_1}{10^1} + \frac{F_2}{10^2} + \frac{F_3}{10^3} + \frac{F_4}{10^4} + \ldots \right)$$

$$R = 100 \left( \frac{F_2}{10^3} + \frac{F_3}{10^4} + \frac{F_4}{10^5} + \frac{F_5}{10^6} + \ldots \right) - 10 \left( \frac{F_1}{10^2} + \frac{F_2}{10^3} + \frac{F_3}{10^4} + \frac{F_4}{10^5} + \ldots \right)$$

$$R = 100 \left( R - \frac{F_0}{10^1} - \frac{F_1}{10^2} \right) - 10 \left( R - \frac{F_0}{10^1} \right)$$

$$R = 90R - 1$$

$$F_0 = 0, F_1 = 1, \quad F_n + F_{n+1} = F_{n+2}$$

$$R = \frac{F_0}{10^1} + \frac{F_1}{10^2} + \frac{F_2}{10^3} + \frac{F_3}{10^4} + \dots$$

$$R = \frac{(F_2 - F_1)}{10^1} + \frac{(F_3 - F_2)}{10^2} + \frac{(F_4 - F_3)}{10^3} + \frac{(F_5 - F_4)}{10^4} + \dots$$

$$R = \left( \frac{F_2}{10^1} + \frac{F_3}{10^2} + \frac{F_4}{10^3} + \frac{F_5}{10^4} + \dots \right) - \left( \frac{F_1}{10^1} + \frac{F_2}{10^2} + \frac{F_3}{10^3} + \frac{F_4}{10^4} + \dots \right)$$

$$R = 100\left( \frac{F_2}{10^3} + \frac{F_3}{10^4} + \frac{F_4}{10^5} + \frac{F_5}{10^6} + \dots \right) - 10\left( \frac{F_1}{10^2} + \frac{F_2}{10^3} + \frac{F_3}{10^4} + \frac{F_4}{10^5} + \dots \right)$$

$$R = 100\left( R - \frac{F_0}{10^1} - \frac{F_1}{10^2} \right) - 10\left( R - \frac{F_0}{10^1} \right)$$

$$R = 90R - 1 \quad \Rightarrow \quad R = \frac{1}{89}$$

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| | | | | | | | | 1 | 3 | | | | | | | | |
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| | | | | | | | | | | 3 | 4 | | | | | | |
| | | | | | | | | | | | 5 | 5 | | | | | |
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0.01123595505617977528089887640449438202247191