Rulers, Ropes & Telescopes

The bargeman's problem

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The bargeman's e-mail

“I have 5 lengths of rope:

- 1 x 10 metres
- 2 x 12 metres
- 2 x 40 metres

I want to be able to cut the ropes into pieces of different lengths and to be able to tie combinations of these together to make longer lengths.

Is there a formula to obtain the optimum number and lengths of pieces ropes (i.e. the minimum number of pieces of ropes to give the most possible combinations of lengths of rope!). The minimum length of rope I need is 6m.”

Continuity error

You can make any shorter length...

...if you allow infinitely many cuts, knots and infinitesimal lengths.

Write the number in binary:

$$\frac{1}{3} = \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \ldots = 0.010101\ldots$$

Back to reality

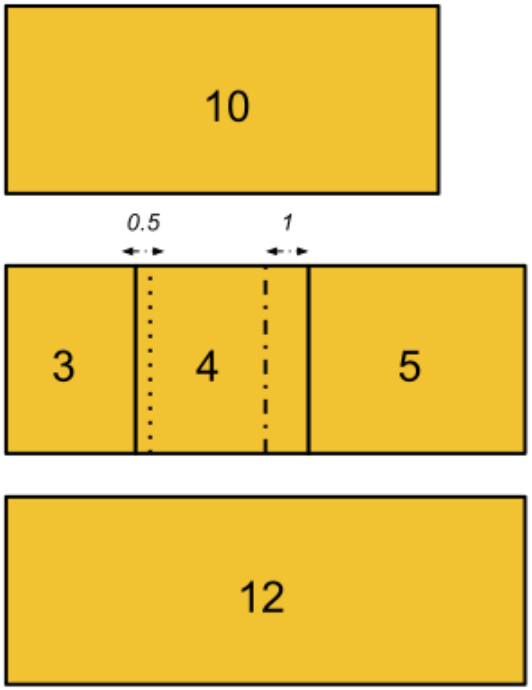
Constraints:

- Integer lengths only (simplicity)
- At most one knot

Modified strategy:

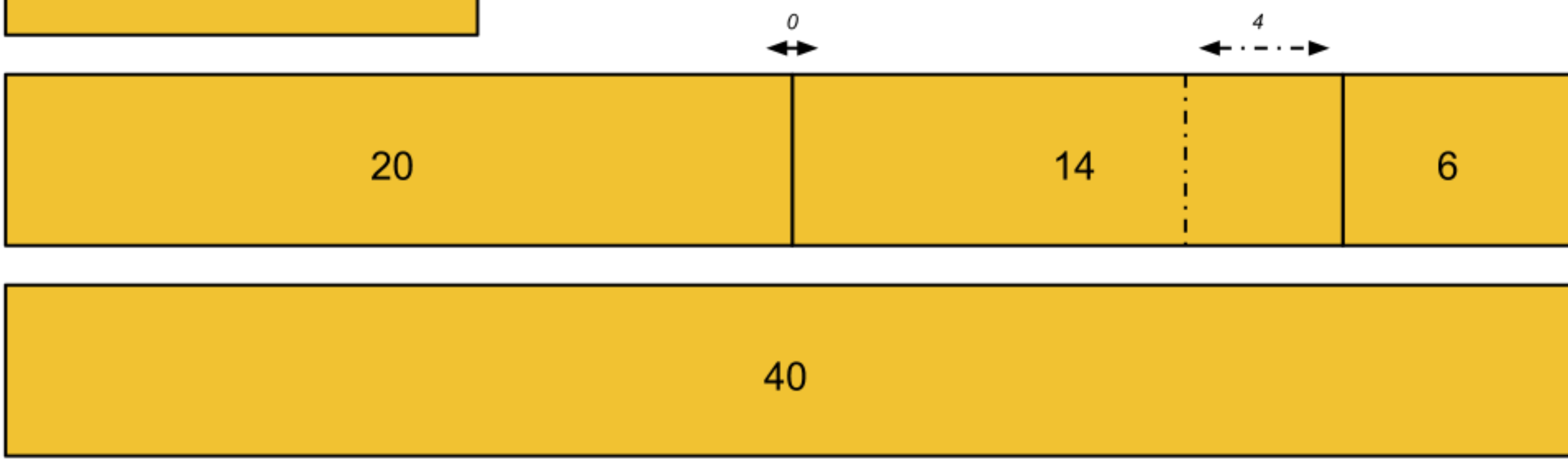
Approximately cut the ropes in half.

Use varying approximations.



These 9 pieces make 35 different lengths out of a maximum 45 $= \binom{9}{2} + 9$

Including every length from 3m to 20m.



No repetitions...

If you add a two unrealistically strict rules:

1. No length can be made in two different ways

2. No length is twice a single rope's length.

You would be looking for **Sidon sets**:

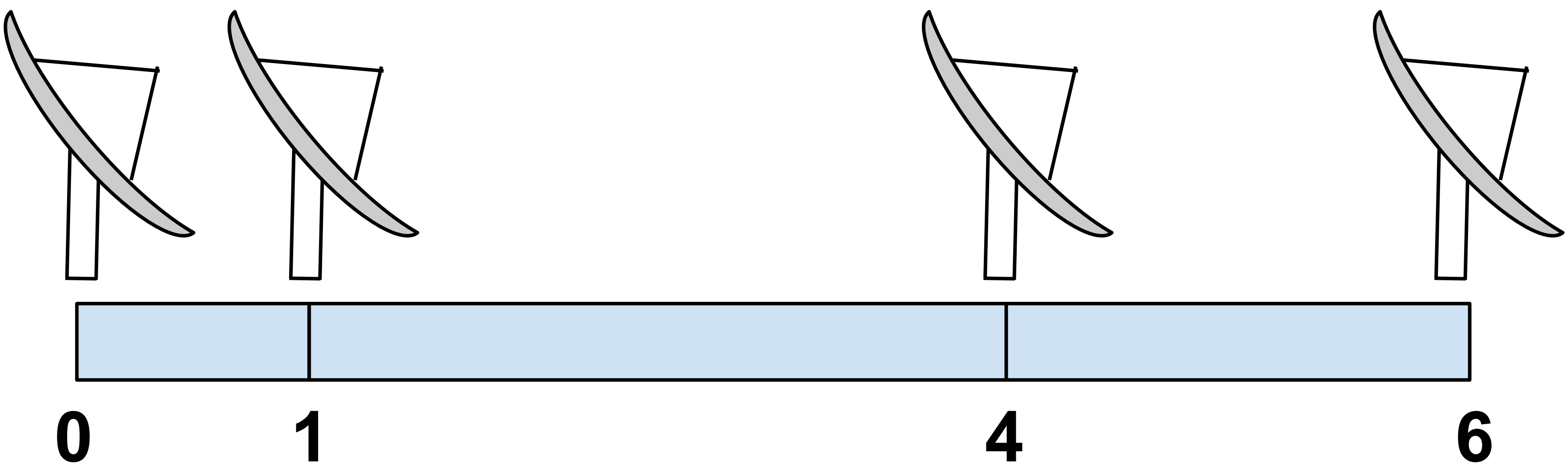
Set of distinct integers $\{a_0 = 0, a_1, \dots a_n\} \subseteq \mathbb{N}$

such that all pairwise sums $a_i + a_j \ (i \leq j)$

are different.

Telescoping series

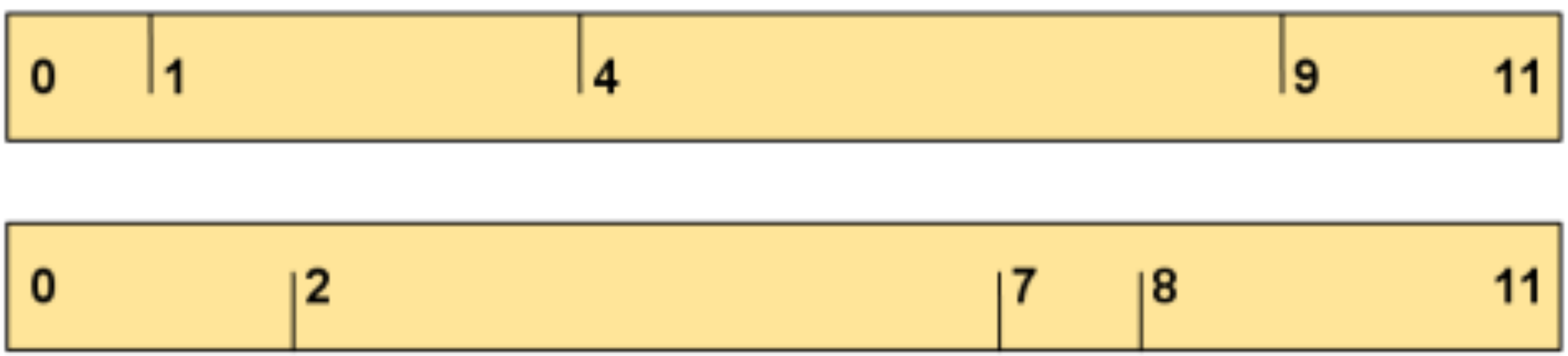
Linear array of radio telescopes find stars by measuring phase difference of light. Want as many differences as possible to improve accuracy.



Golomb rulers

Rulers with so few markings, that any measurable length can only be measured in one way.

This Golomb ruler can't measure 6.



This Golomb ruler can't measure 10. So neither is 'perfect'.

Tying it all together

(Dimitromanolakis, 2002)

All Sidon sets (and overly strict bargeman ropes) are Golomb rulers, and visa-versa.

One-line proof, essentially:

$$a + b \neq c + d \iff a - d \neq c - b$$

Similarly, Bargeman ropes have fewer repeats than any ruler (or telescope array) using the same numbers.