

Why do a *capella* singers go flat...?

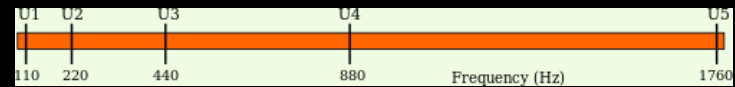
The mathematics of tuning
systems in music

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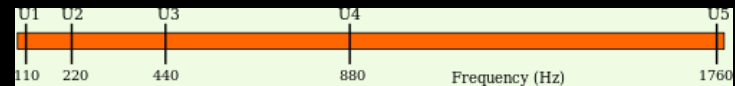
Some points to consider

- Division of the octave



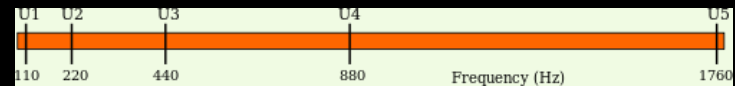
Some points to consider

- Division of the octave
- Number of pitches to be used



Some points to consider

- Division of the octave
- Number of pitches to be used
- Tuning of the pitches



Western tuning systems

- Equal temperament
- Based on dividing into 12 equal parts

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- Pythagorean tuning
- Based on dividing into 12 equal parts
- Based on 'perfect fourths' and 'fifths'

Western tuning systems

- Equal temperament
- Pythagorean tuning
- Just intonation
- Based on dividing into 12 equal parts
- Based on 'perfect fourths' and 'fifths'
- Based on whole-number ratios

Fundamentally harmonic...

Just intonation likes the lower end of the series

The diagram shows two musical staves. The top staff is in bass clef and contains 12 notes. Brackets above the notes indicate intervals: a subminor third between notes 6 and 7, a supermajor second between notes 7 and 8, a greater tone between notes 8 and 9, and a lesser tone between notes 9 and 10. The bottom staff is also in bass clef and contains 12 notes. Brackets below the notes indicate intervals: a perfect fifth between notes 2 and 3, a perfect fourth between notes 6 and 8, and an octave between notes 4 and 12. The notes are numbered 1 through 12 below each staff.

subminor third supermajor second

greater tone lesser tone

1 2 3 4 5 6 7 8 9 10 11 12

Octave

Perfect fifth Perfect fourth

1 2 3 4 6 8 12

Pythagorean tuning only likes octaves, fifths and fourths

A matter of ratios...

- $5/12 = 5/6 = 5/3 = 10/3 = 20/3$

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Get used to it, if you want to read about tuning theories!

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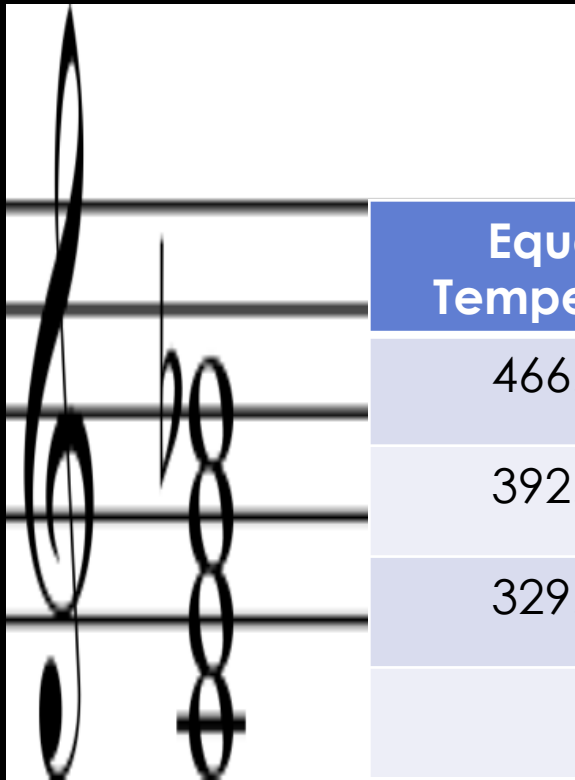
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A musical staff with a treble clef and five lines. It shows five notes: a whole note on the first line (C), a whole note on the second line (G), a whole note on the third space (D), a whole note on the fourth space (A), and a whole note on the fifth space (E). Above the staff, the ratios for the intervals are given: 3/2 between C and G, 9/4 between C and D, 27/8 between C and A, and 81/64 between C and E. Below the staff, the intervals are labeled: Perfect fifth, fifth fourth, fifth fourth, and fourth. A text box at the top right of the staff area says "Uh-oh, just intonation ratio is 5/4".

Interval	Ratio
Perfect fifth	3/2
Fifth fourth	9/4
Fifth fourth	27/8
Fourth	81/64

Uh-oh, just intonation ratio is 5/4

What to choose?



Equal Tempered	Just intoned	Pythagorean tuning
466.2	$7/4 = 457.8$	$16/9 = 465.1$
392.0	$3/2 = 392.4$	
329.6	$5/4 = 327.1$	$81/64 = 331.1$
	261.6	

Frequencies in Hertz based on A = 440Hz

Now the fun begins!

Just
5:4

Needs to be adjusted
by syntonic comma
81/80, otherwise pitch drops!

Fundamental is C

Fundamental Fundamental Fundamental Fundamental Fundamental
E A D G C

Now the fun begins!

$$\frac{5}{4} \times \frac{81}{80} = \frac{81}{64}$$

Just 5:4

Needs to be adjusted by syntonic comma 81/80, otherwise pitch drops!

Fundamental is C Fundamental E Fundamental A Fundamental D Fundamental G Fundamental C

Uh-oh, just intonation ratio is 5/4

3/2 9/4 27/8 81/64

Perfect fourth fifth fourth fifth fourth