# Why do *a capella* singers go flat…?

The mathematics of tuning systems in music

Colin Graham

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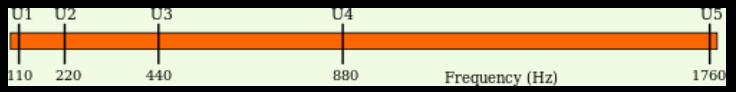
# Some points to consider

- Division of the octave



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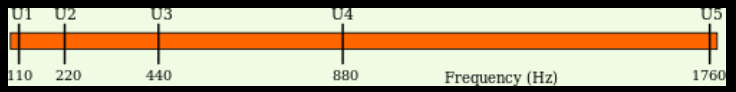


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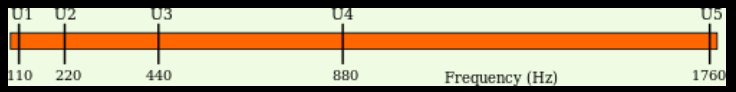
- Division of the octave
- Number of pitches to be used
- Tuning of the pitches

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- Just intonation

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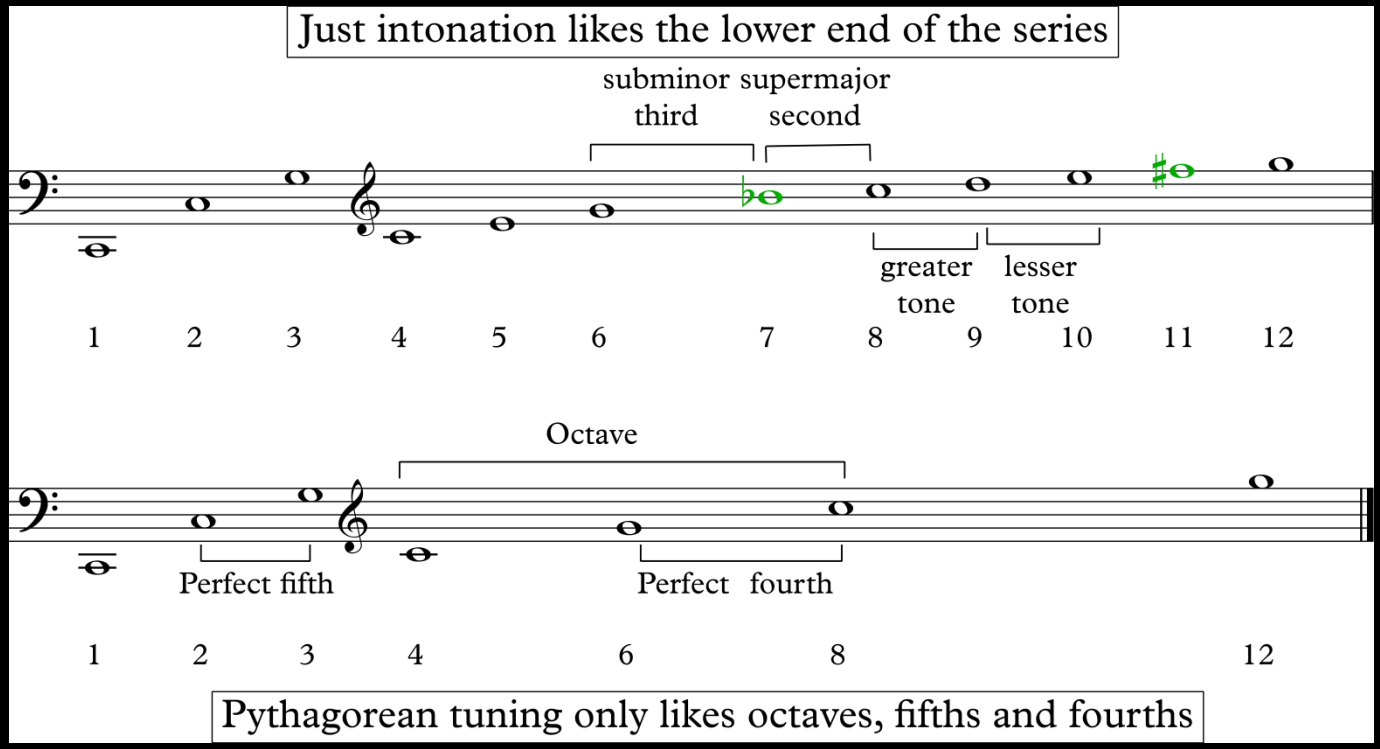
- Based on ‘perfect fourths’ and ‘fifths’
- Based on whole-number ratios

# Fundamentally harmonic…



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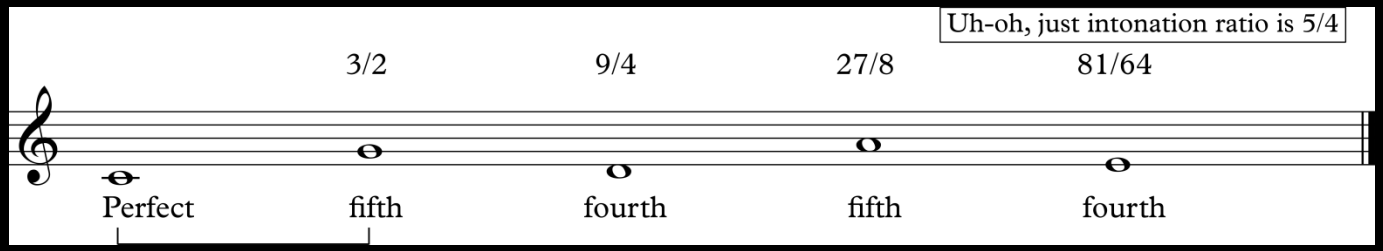
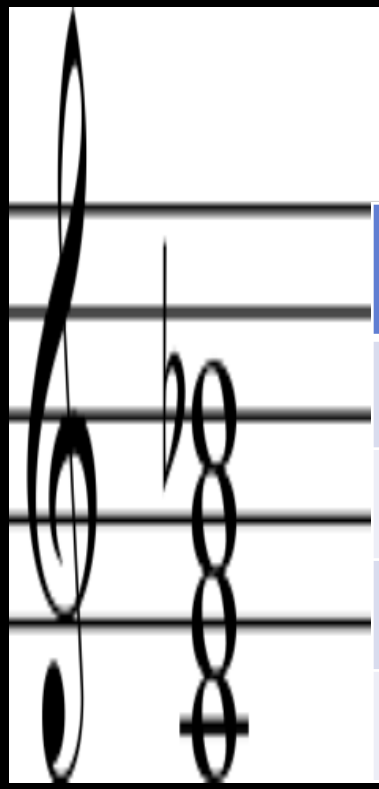


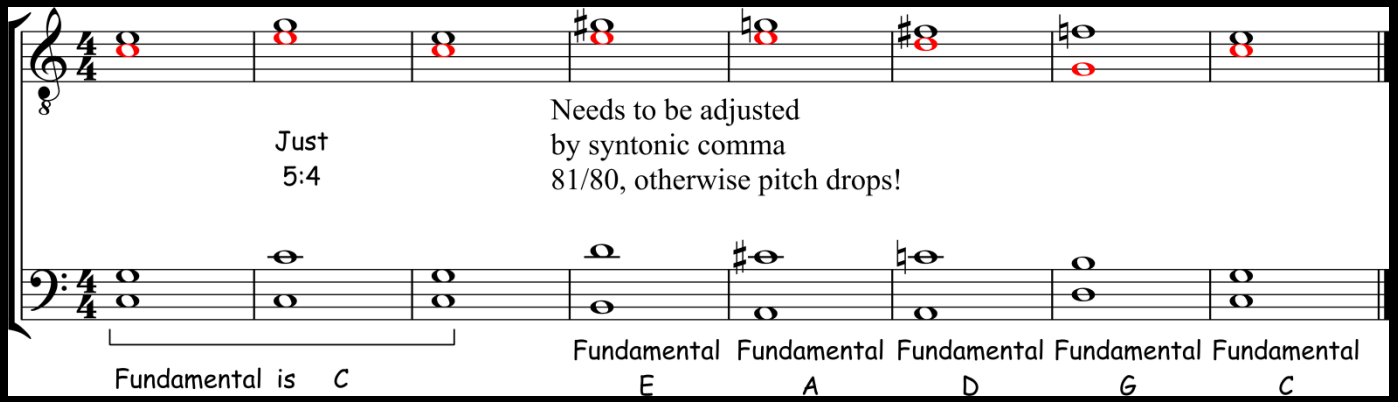
# What to choose?



| Equal Tempered | Just intoned | Pythagorean tuning |
|---|---|---|
| 466.2 | 7/4 = 457.8 | 16/9 = 465.1 |
| 392.0 | 3/2= 392.4 | |
| 329.6 | 5/4 = 327.1 | 81/64 = 331.1 |
| 261.6 | | |

Frequencies in Hertz based on A = 440Hz

# Now the fun begins!



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$$\frac{5}{4} \times \frac{81}{80} = \frac{81}{64}$$