Approximating log 3 using a piano or "$3 = 10^{what}$"

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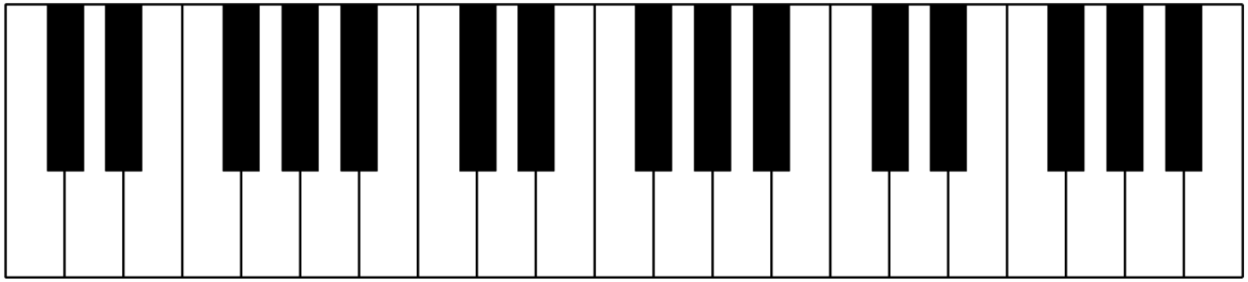
Approximating log 3 using a piano

- $2^{10} \approx 10^3$ so $2 \approx 10^{3/10}$ i.e. $\log 2 \approx 3/10 = 12/40$



- An **octave** → 2 times the frequency
- A **semitone** (1/12 octave) → $2^{1/12} \approx 10^{1/40}$
- A **fifth** (7 semitones) → $10^{7/40}$ times the frequency

BUT a **fifth** is also about 3/2 times the frequency:

$$3/2 \approx 10^{7/40}$$

$$\log 3 - \log 2 \approx 7/40$$

$$\mathbf{\log 3} \approx 7/40 + \log 2 = 19/40 = \mathbf{0.475}$$

(actually 0.477 to 3dp)

log 5?

We have log 5 = log 10 – log 2 ≈ 0.7, but the piano works too:

A **major third** (4 semitones) → $10^{4/40}$ times the frequency

BUT a **major third** is also about 5/4 times the frequency:

$5/4 \approx 10^{1/10}$

log 5 – 2 log 2 ≈ 1/10

log 5 ≈ 1/10 + 2 log 2 = 7/10 = **0.7**

(actually 0.699)

How many times do we need to quintuple our chair
to have 10 million = 10^7 chairs?
Answer about 7/0.7 = 10.