Washing up after the potluck dinner

A question about Joe’s puzzlingly particular parties

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Joe’s particular potluck dinners

- Everyone brings a different number of portions.
- Everyone brings enough portions for everyone to have at least one of each dish.

Joe enforces the rules by handing out storage containers, each holding a different number of portions.

Washing up after one such dinner, Joe takes out the largest container of size **n**.

How many different sets of containers could Joe be washing up?

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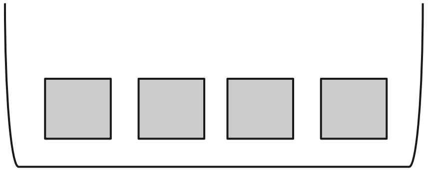

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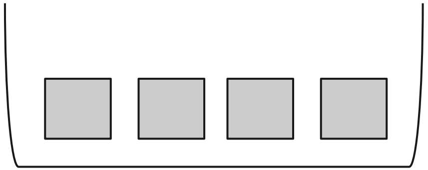
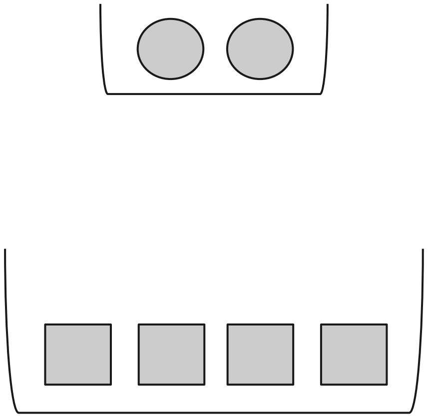


2: {2,4}



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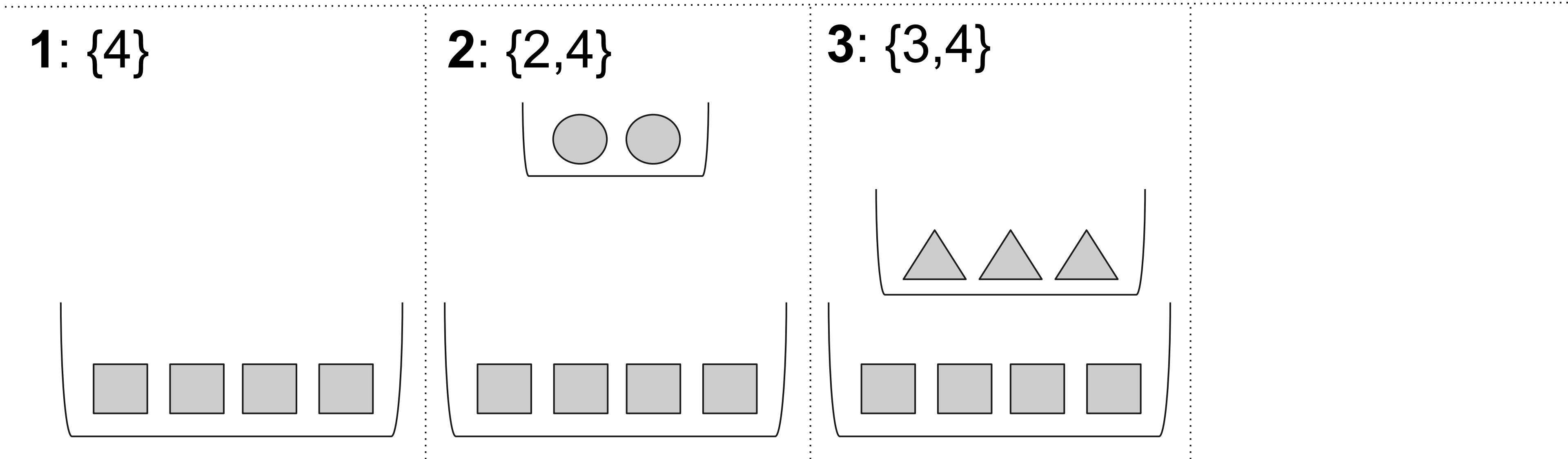

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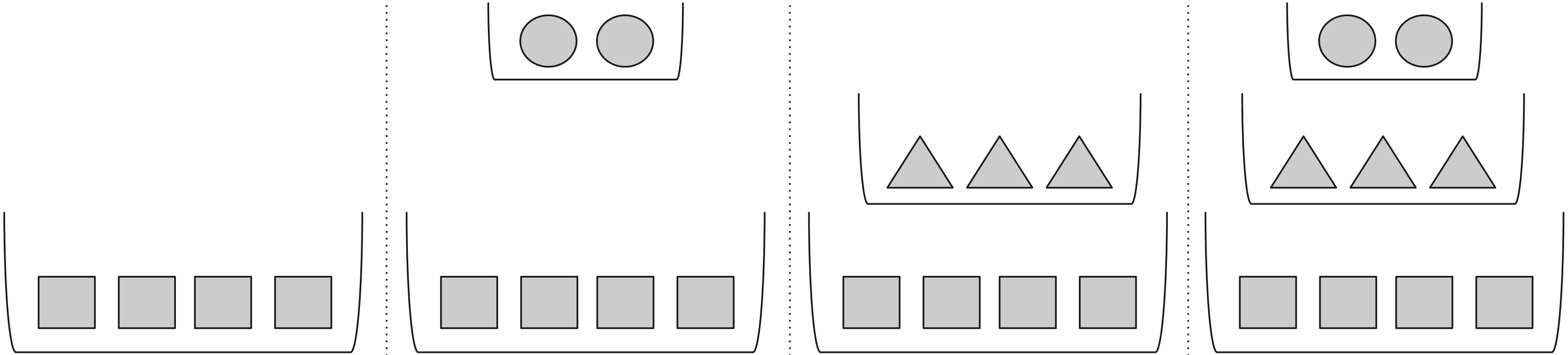

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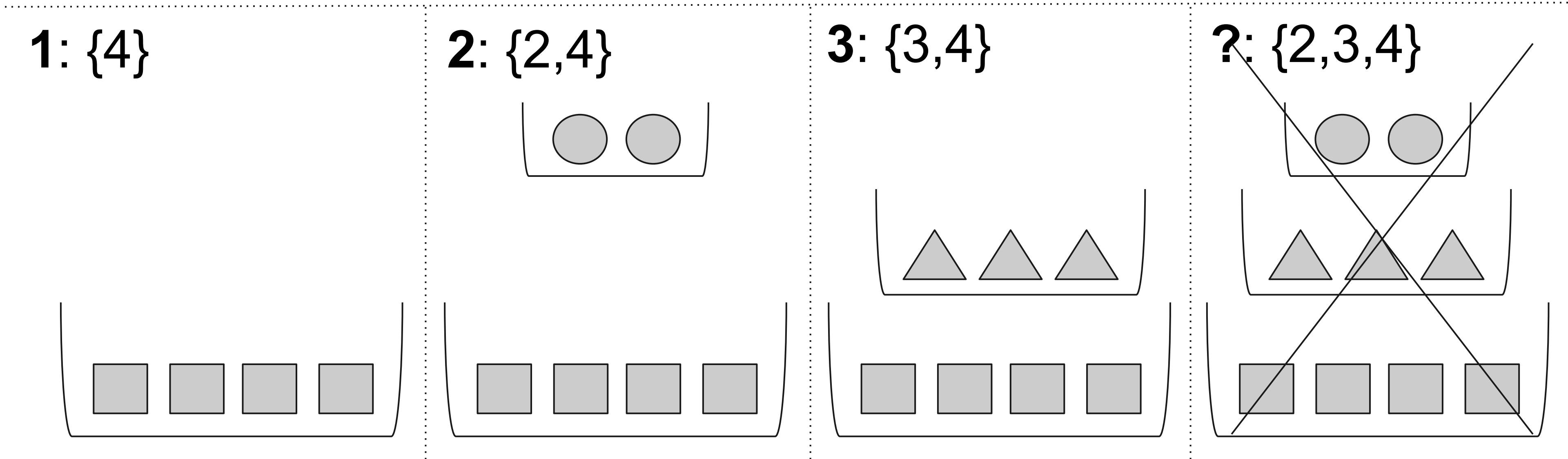

Schreier sets

- A set S of natural numbers is a Schreier set if:

$$|S| \leq \min(S).$$

- Introduced in 1930 by Josef Schreier.
- Independently discovered in 1982 by Ramsey theorists.

Die Klammern

$(3, 4,), (3, 5,)\ (3, 6,) \ldots, (3, \infty),$ (1)
$(4, 5, 6), (4, 5, 7) \ldots, (4, 5, \infty), (4, 6, 7) \ldots (4, 6, \infty) \ldots (4, \infty, \infty),$
$(5, 6, 7, 8) \ldots (5, 6, 7, \infty), (5, 6, 8, \infty) \ldots (5, 6, \infty, \infty) \ldots (5, 7, \infty, \infty), (5, \infty, \infty, \infty),$
$(6, 7, 8, 9, 10) \ldots\ldots$

bilden in der hingeschriebenen Ordnung eine wohlgeordnete Menge vom Typus ω^{ω}.

(Man sieht, daß in (1) jede Klammer vorkommt, die mit einer natürlichen Zahl $k+1$ beginnt und k natürliche Zahlen enthält:

$$k+1 = n_{k+1} < n_{k+2} < \ldots\ldots\ \ldots n_{2k}).$$

for each $\alpha \geqslant \omega$. A typical example of an ω-uniform system is the set of all $s \subseteq X$ such that the cardinality of s equals to the least element of s. Similarly for $\omega + 1$,

- **Question re-stated**: How many Schreier sets are there with maximal element **n**?

Thanks to Ron Knott for his idea to present this puzzle as potluck dinners.