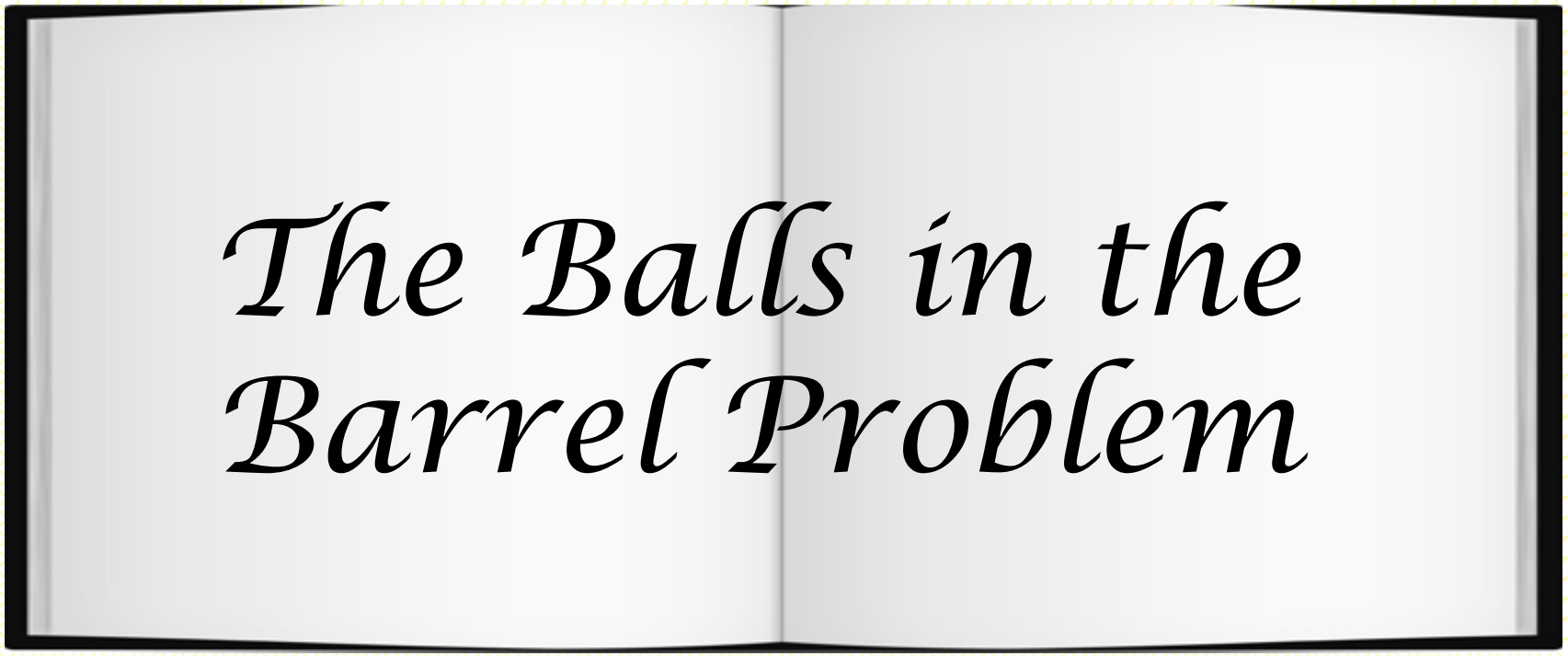
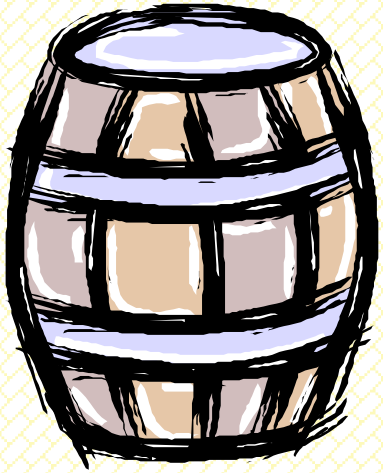



David Bedford

# Balls in Barrels

A barrel contains infinitely many balls numbered 1,2,3,4,…

The hour from 11pm to midnight is divided into infinitely many intervals $T_1$, $T_2$, $T_3$, …



# Balls in Barrels

$T_1$: take out balls number 1 and 2

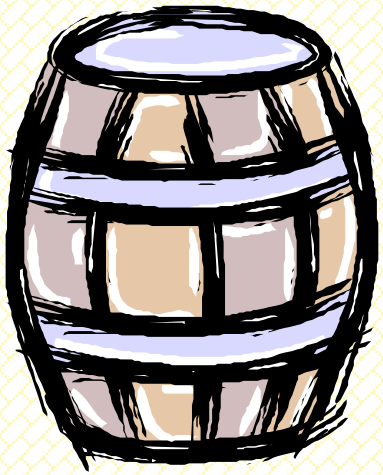
$T_2$: replace ball number 1

$T_3$: take out balls number 3 and 4

$T_4$: replace ball number 3

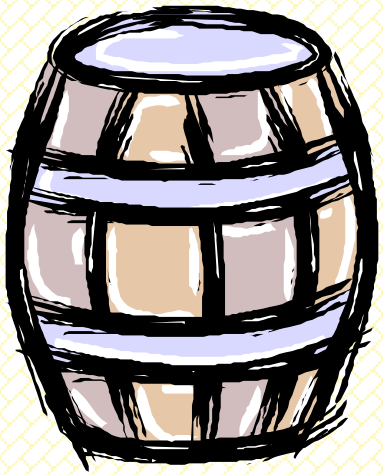
$T_5$: take out balls number 5 and 6

$T_6$: replace ball number 5 and so on…



# At 1 minute past midnight…

How many balls are outside the barrel?

All the odd numbered balls are in the barrel and all of the even numbered balls are outside.

So, as we'd expect (?), there are infinitely many balls outside the barrel.

Let's try again with a different strategy

Once again the hour from 11pm to midnight is divided into infinitely many intervals $T_1$, $T_2$, $T_3$, …

$T_1$: take out balls number 1 and 2

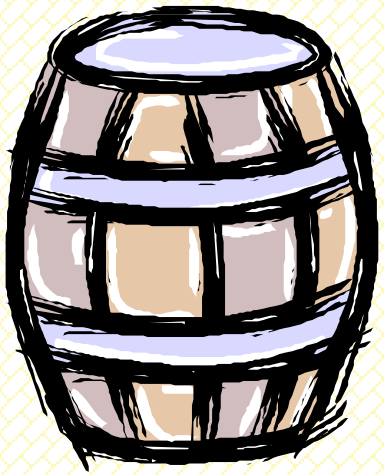
$T_2$: replace ball number 1

$T_3$: take out balls number 3 and 4

$T_4$: replace ball number 2

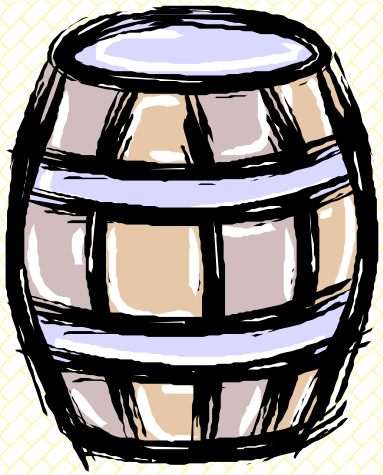
$T_5$: take out balls number 5 and 6

$T_6$: replace ball number 3 and so on…



# At 1 minute past midnight…


How many balls are outside the barrel?



This time every ball is taken out once AND every ball is replaced once.

This time there are no balls outside the barrel.

Let's try again with a different strategy

Once again the hour from 11pm to midnight is divided into infinitely many intervals $T_1$, $T_2$, $T_3$, …

$T_1$: take two balls out

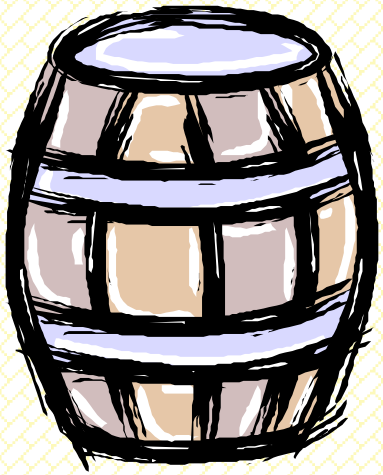
$T_2$: put one ball back

$T_3$: take two balls out

$T_4$: put one ball back

$T_5$: take two balls out

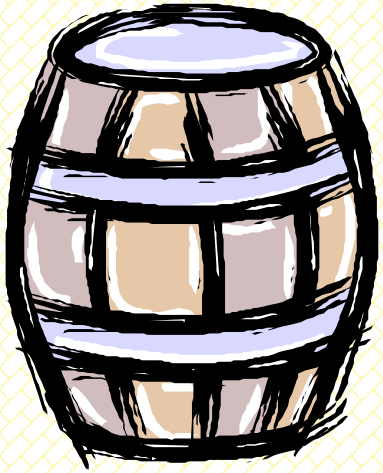
$T_6$: put one ball back and so on…



# At 1 minute past midnight…

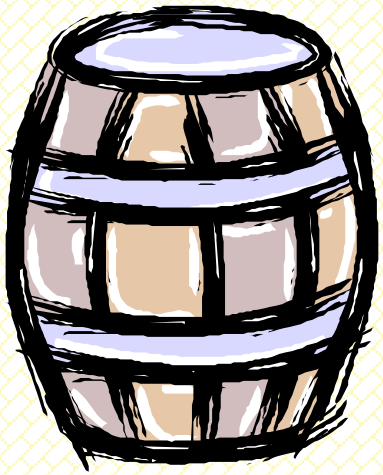

How many balls are outside the barrel?

How many would you like?



Let's try again with numbered balls

This time we choose the ball to be replaced uniformly at random



$T_1$: take out balls number 1 and 2

$T_2$: pick one at random and replace

$T_3$: take out balls number 3 and 4

$T_4$: pick one (out of 3) at random and replace

$T_5$: take out balls number 5 and 6

$T_6$: pick one (out of 4) at random and replace and so on…

# At 1 minute past midnight…


How many balls are outside the barrel?

Obviously 0 or $\infty$

They all end up back in the barrel.