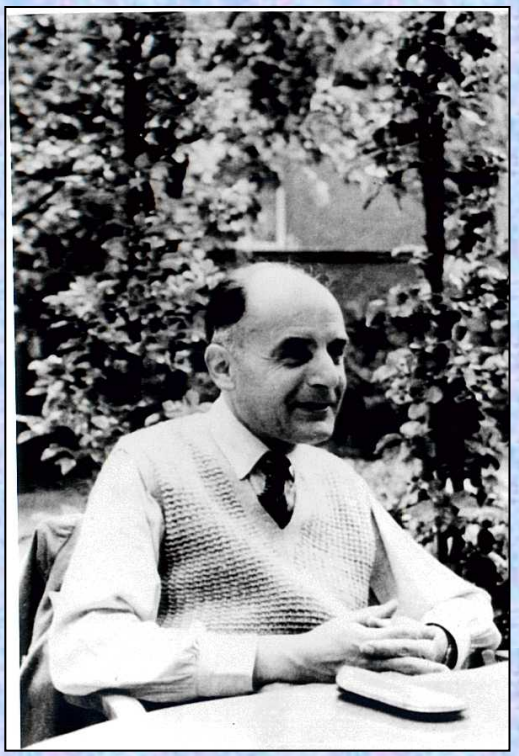
MathsJam 2015

A paradox resolved?

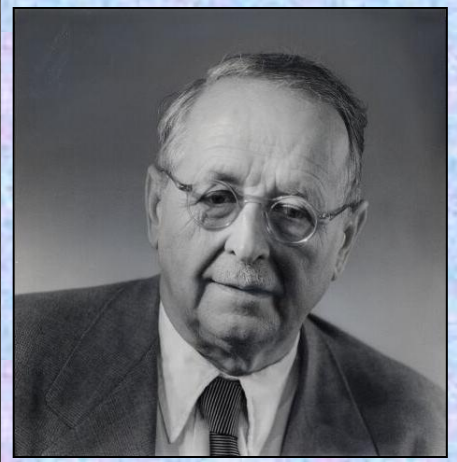
Formulated by Grelling and Nelson (1908)

Wrongly called Weyl's Paradox



Kurt Grelling

Leonard Nelson



Herman Weyl

Self-descriptive

Unhyphenated

Pentas

Hellenic

Sesquipedalian

Seventeen-lettered

Antepenultimate

Dactylic

Puzzling

Heterological

Green

Non-self-descriptive

Long

Phonetic

Green

Neological

Infinite

Palindromic

Aforementioned

Pronounceable

Axiom 1. $\{P(\varphi) \wedge \Box \forall x\,[\varphi(x) \rightarrow \psi(x)]\} \rightarrow P(\psi)$

Axiom 2. $P(\neg\varphi) \leftrightarrow \neg\, P(\varphi)$

Theorem 1. $P(\varphi) \rightarrow \Diamond \exists x\,[\varphi(x)]$

Definition 1. $G(x) \Leftrightarrow \forall \varphi\,[\,P(x) \rightarrow \varphi(x)]$

Axiom 3. $P(G)$

Theorem 2. $\Diamond \exists x\, G(x)$

Definition 2. $\varphi \text{ ess } x \Leftrightarrow \varphi(x) \wedge \forall \psi\{\psi(x) \rightarrow \Box \forall y[\varphi(y) \rightarrow \psi(y)]\}$

Axiom 4. $P(\varphi\,) \rightarrow \Box\, P(\varphi)$

Theorem 3. $G(x) \rightarrow G \text{ ess } x$

Definition 3. $E(x) \Leftrightarrow \forall \varphi[\varphi \text{ ess } x \rightarrow \Box \exists y\, \varphi(y)]$

Axiom 5. $P(E)$

Theorem 4. $\Box \exists x\, G(x)$

That’s all folks