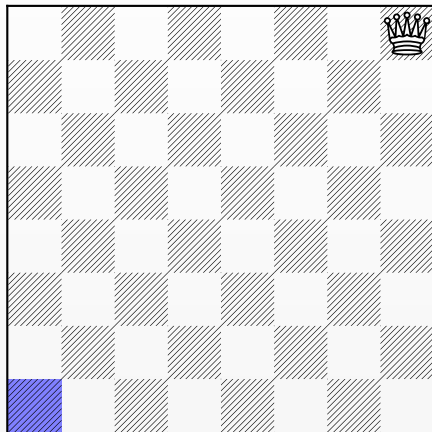
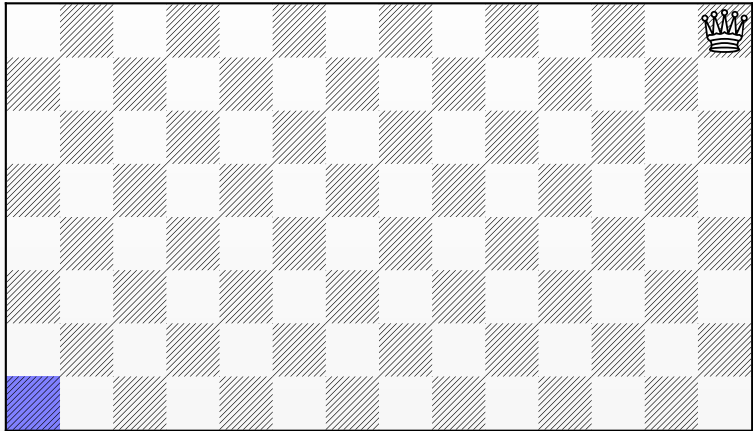


Before I start



Before I start



- ▶ Players take turns to move the Queen west, south or southwest.
- ▶ Player to get Queen to blue square first wins.

Nim-like games

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Maths Jam Conference VI

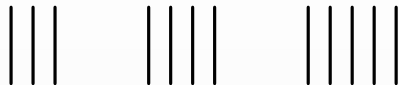
7th November 2015

Nim

- ▶ A game for two players.
- ▶ Players take turns to remove objects from several piles, subject to some rules.
- ▶ The last player to remove an object is the winner (or, sometimes, the loser).

Typical Nim example

- ▶ Say we start with three piles of matches of size 3, 4 and 5.
- ▶ Players may remove as many matches as they like from a single pile each turn.
- ▶ Last player to remove a match wins.



How to solve it — illustration

- ▶ Say we have three piles of one match per pile.



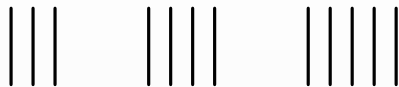
- ▶ If I remove one pile, you must remove another and I can remove the third, meaning I win.

- ▶ Looking at the xor sum of the binary representation of the three piles:
 $1 \oplus 1 \oplus 1 = 1.$

- ▶ A winning move is to leave the xor sum at zero at the end of my go, e.g. here:
 $0 \oplus 1 \oplus 1 = 0.$

Typical Nim example

- ▶ So the Nim-sum here is:
 $011 \oplus 100 \oplus 101 = 010$.



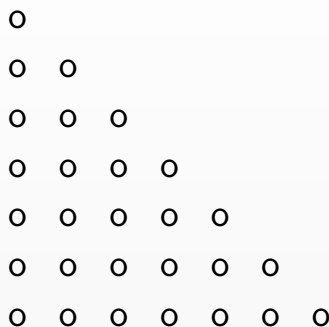
- ▶ A winning move would be to remove 2 matches from the pile of 3:
 $001 \oplus 100 \oplus 101 = 000$.



- ▶ Play it through!

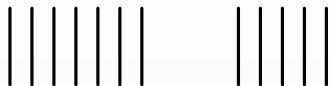
Solomon's Stones

- ▶ A game we use at the Maths Arcade.
bit.ly/mathsarcade
- ▶ Remove any number of counters from any row OR column.
- ▶ Like overlapping multi-pile Nim.
- ▶ Studied by my project student Anthony Fox last year.



Wythoff's Game

- ▶ Two piles, an arbitrary number of matches in each pile.
- ▶ Players take turns; valid moves are:
 - ▶ remove at least one from either pile;
 - ▶ remove at least one from both piles provided the same number are removed from each pile.
- ▶ Player who takes the last counter wins.

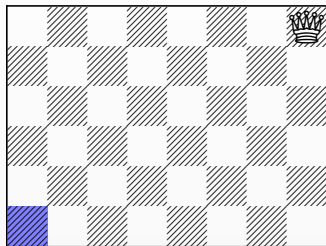


A nice thing about Nim-like games...

Wythoff's Game

is...

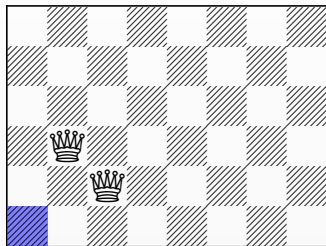
- ▶ Two piles, an arbitrary number of matches in each pile.
- ▶ Players take turns; valid moves are:
 - ▶ remove at least one from either pile;
 - ▶ remove at least one from both piles provided the same number are removed from each pile.
- ▶ Player who takes the last counter wins.



- ▶ Players take turns to move the Queen west, south or southwest.
- ▶ Player to get Queen to blue square first wins.

Wythoff's Game as a chessboard

- ▶ 'Safe' square: if you leave the Queen there, your opponent is forced to put you in a winning position (e.g. the two Queens shown to the left are 'safe').
- ▶ Label the blue square as $(0,0)$. These Queens are a symmetric pair with coordinates $(1,2)$ $((1,2)$ and $(2,1))$.



- ▶ The pattern of safe squares is interesting.
- ▶ There's a Martin Gardner article.

Thanks

- ▶ I've shown you Nim, Solomon's Stones and Wythoff's Game.
- ▶ Thank you for listening.
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