# A Schoolboy's Surprise for Professor Tutte

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# SQUARED RECTANGLES

**Squared rectangle (square):** rectangle (square) dissected into a finite number, two or more, of squares.

**Element:** component square or its side-length.

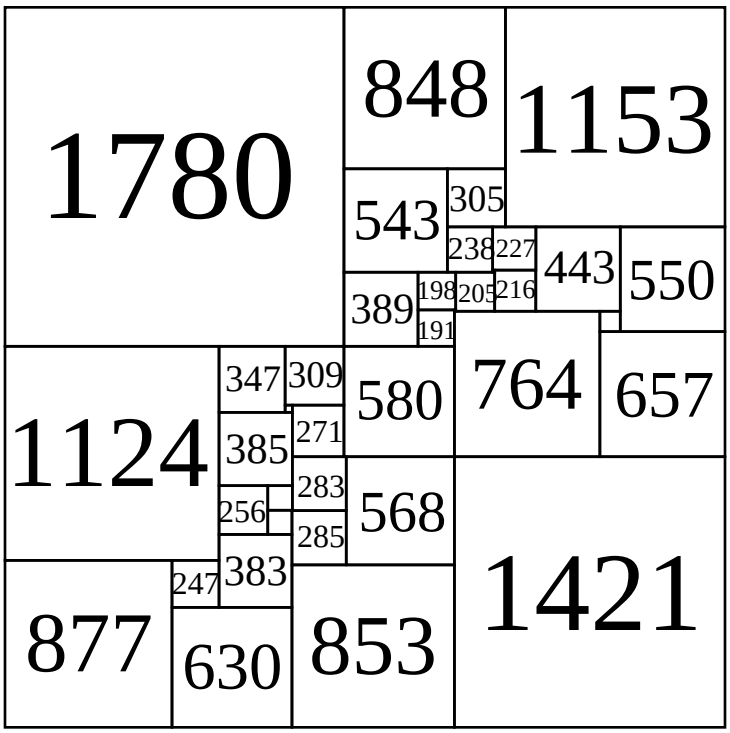
**Order:** number of elements.

A squaring is

- **simple** if it does not contain a smaller squared rectangle, otherwise it is **compound;**
- **perfect** if no two squares are the same size, otherwise it is **imperfect.**

**PSS:** Perfect Squared Square.

# EXAMPLE OF A PSS WITH THREE KINDS OF BLEMISH




39:3781 (GHM,2012)

- Compound: contains a smaller squared rectangle;
- Crossed: that is, four of the component squares meet at a point;
- Has a component square, not one of the four corner squares, bisected by a diagonal of the complete figure.

# SIMPLE PSSs (SPSSs) PRE-1964

**First SPSS:** found by Tutte in 1939[8], perhaps the 55:5468 published by the Four (RLB,CABS,AHS,WTT) in 1940[3].

**First blemish-free PSS:** Tutte's 69:7919535 constructed by complicated means, published in 1950[9]. He missed simpler ways based on what a schoolboy showed him on a blackboard years later.

**Only five other SPSSs were reported before 1964, none of them blemish-free:**

- 55:16176 (CJB 1947[1]);
- 52:182069 (CABS&WTT 1950[2,9]);
- 70:384948 (WTT 1950[9]);
- 38:4920 (RLB 1950[2,9]);
- 37:1947 (THW found 1959[10])
  - published 1962[2,5,12].

# "WITHOUT EVER HAVING SEEN ONE"

1936-1938 at Trinity College, Cambridge:

Undergraduates Leonard Brooks, Cedric Smith, Arthur Stone and Bill Tutte researched the problem of ’Squaring the Square’. Then in 1939 Tutte found the first SPSS.

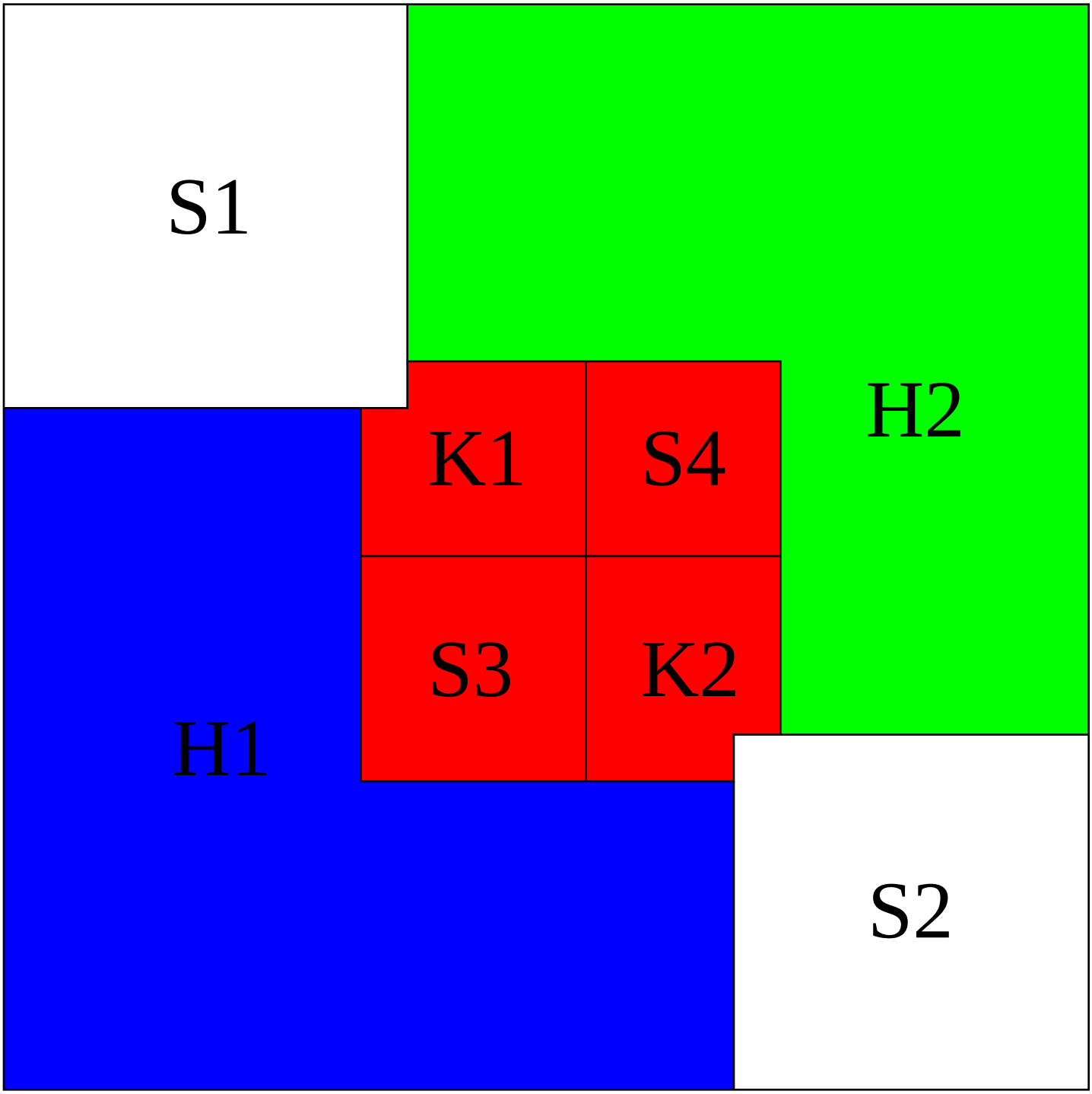
January 1942 at Bletchley Park:

Bill Tutte discovered how the German Lorenz enciphered teleprinter worked **without ever having seen one.** This led to the creation of Colossus.

June 1964 at Wyggeston Boys’ School, Leicester, where Cedric Smith was once a pupil:

A Wyggeston schoolboy showed Brooks, Smith, Tutte, and others a new way to construct SPSSs **without ever having seen one.**

# SCHOOLBOY'S SPSS METHOD

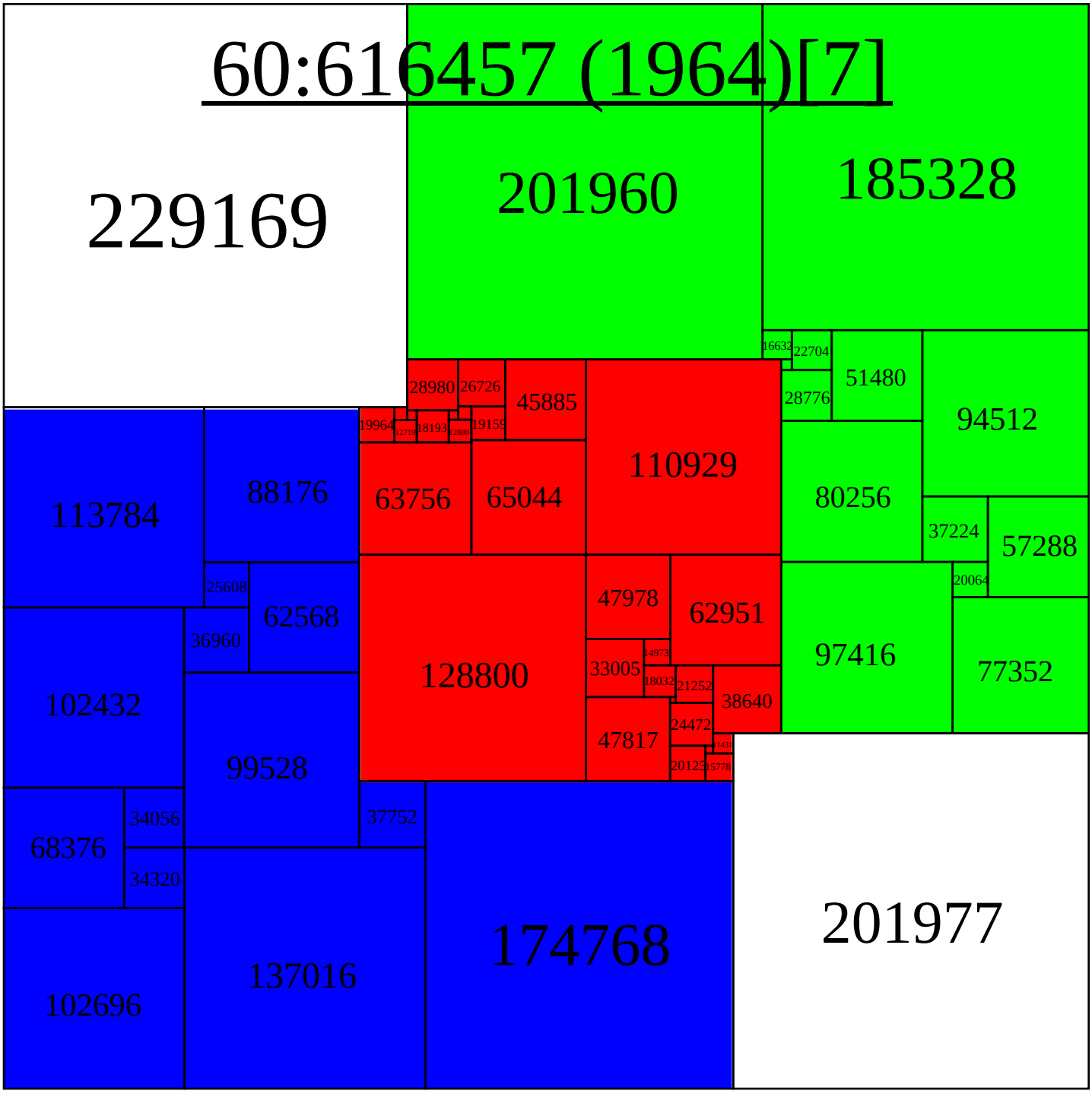



H1, H2 are congruent squared hexagons got by omitting a common corner element from two congruent squared rectangles with no other element in common.
Such pairs of rectangles can be found by the 'rotor-stator' method described by Brooks, Smith, Stone and Tutte.[3,10]

K1, K2 are another such pair, or H1, H2 suitably scaled.

S1-S4 are squares.

# SCHOOLBOY'S FIRST SPSS

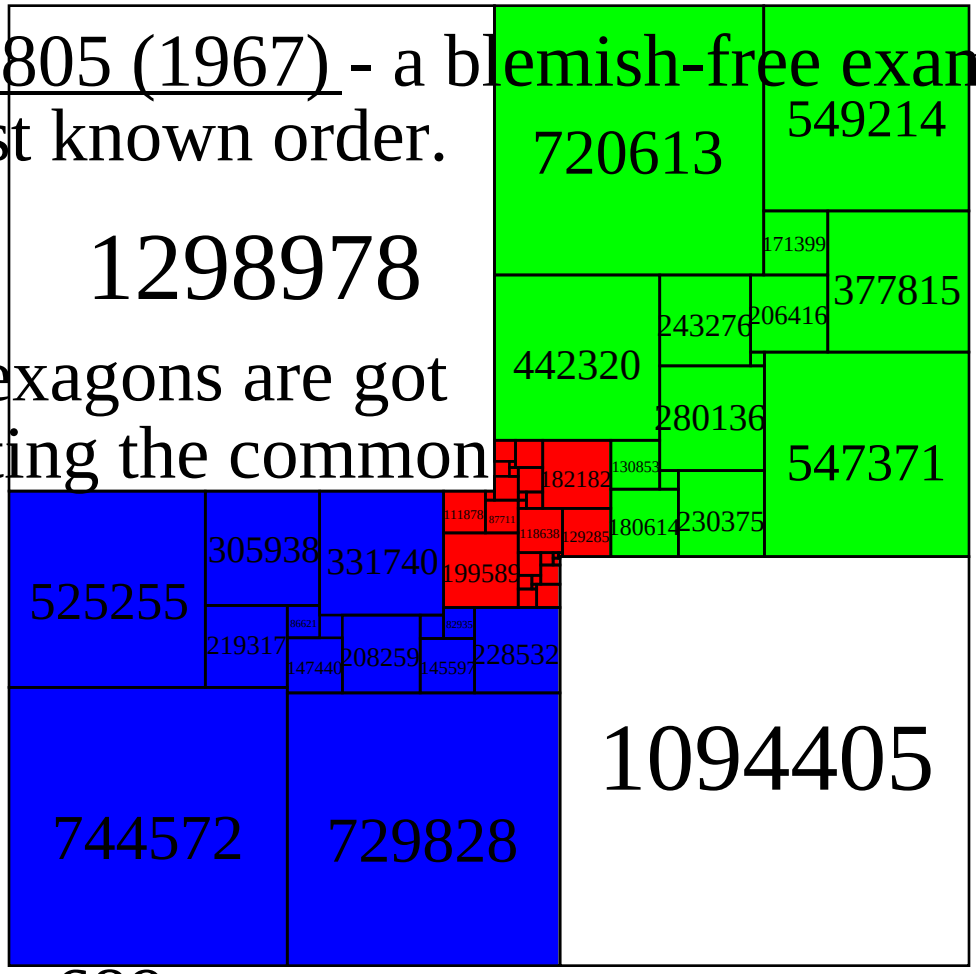



P. J. Federico wrote in an annotated bibliography[6]:

> "A highly ingenious method of constructing simple perfect squared squares of high order, 60 or above, from specially related pairs of squared rectangles, discovered by the author while in high school."

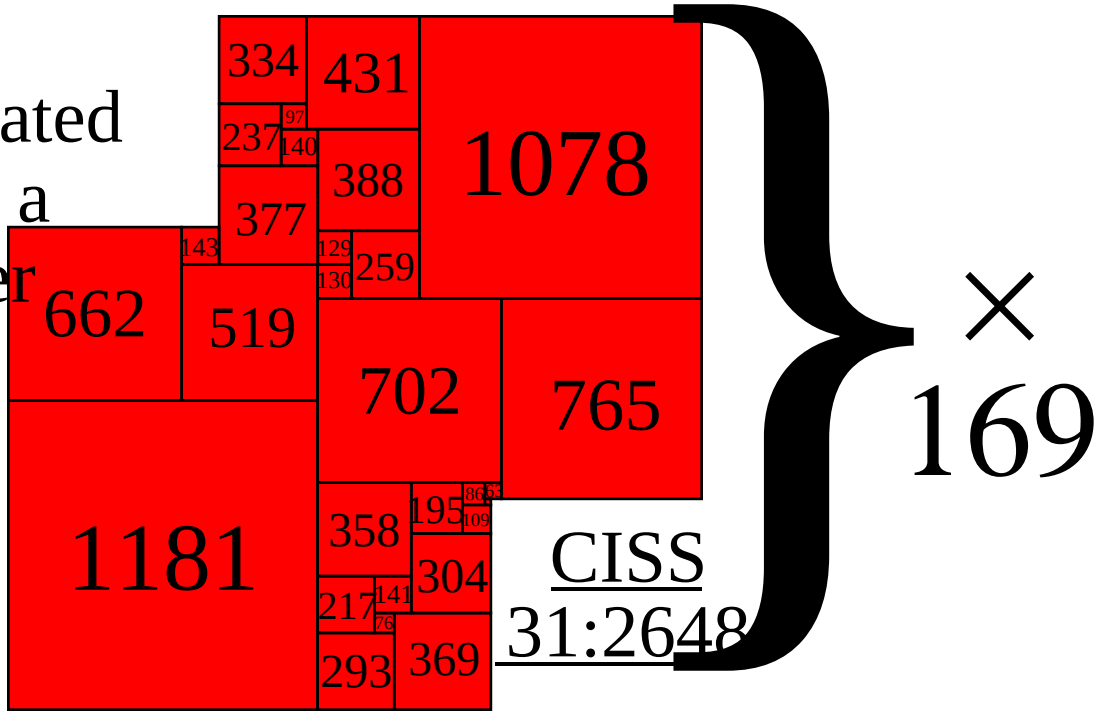
# FIRST GENERALISATION: NO RESTRICTION ON HOW THE OCTAGON IS SQUARED.

59:2568805 (1967) - a blemish-free example of lowest known order.




Outer hexagons are got by omitting the common corner element (169) from a pair of SPSRs 15: 800 x 689.

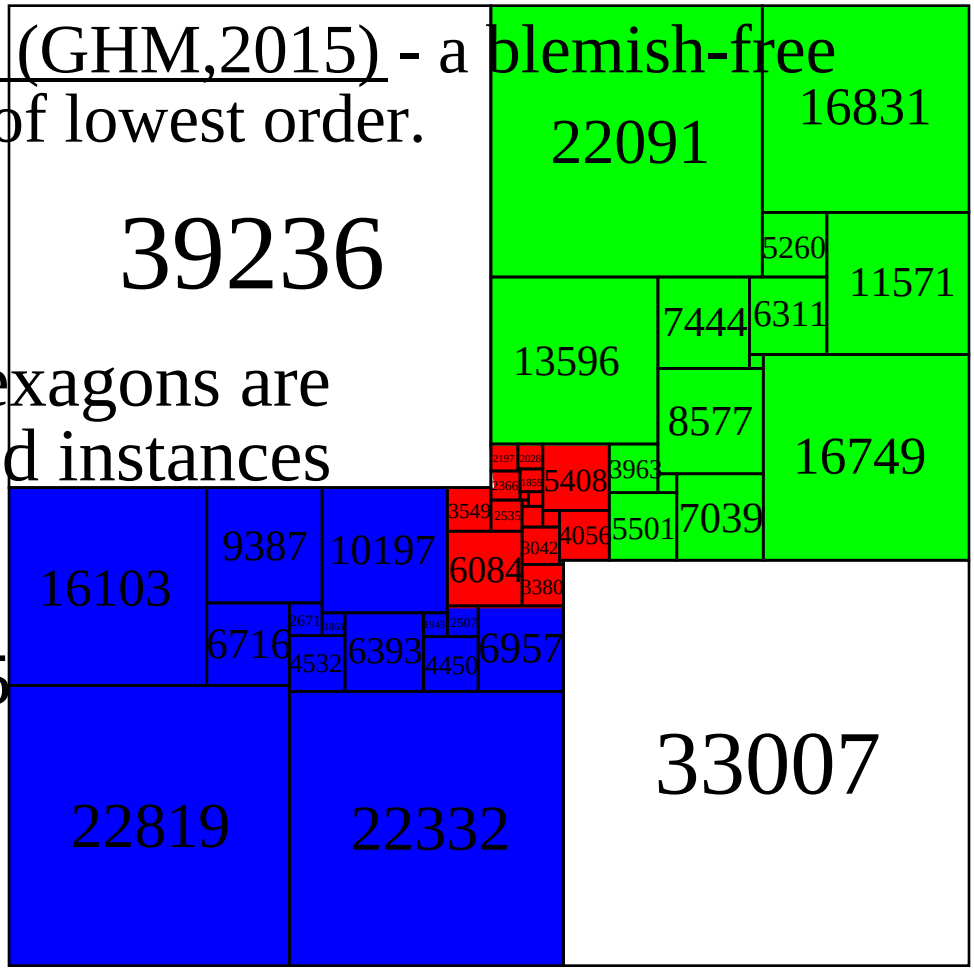
Cross eliminated by removing a second corner square from one of the two small rectangles.

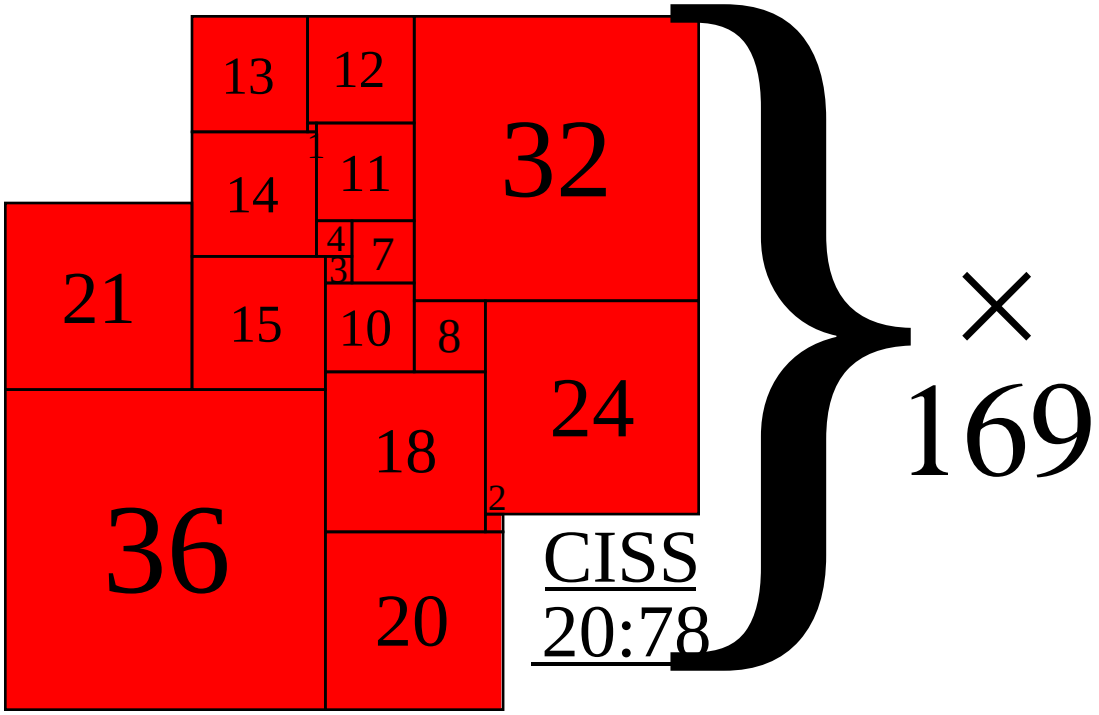


# SECOND GENERALISATION: THE OCTAGON'S MISSING CORNER SQUARES MAY BE DIFFERENT SIZES.

48:78158 (GHM,2015) - a blemish-free example of lowest order.

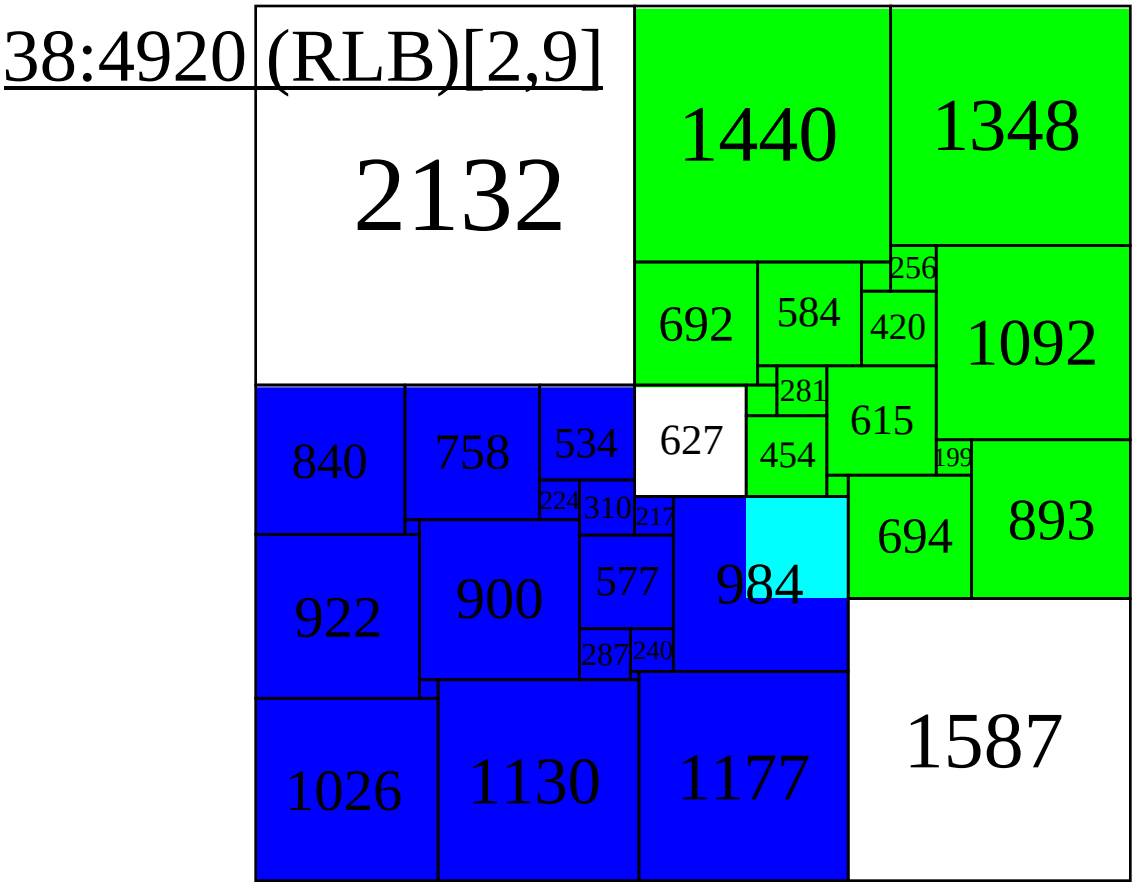
Outer hexagons are deformed instances of those in 59: 2568805 (1967).



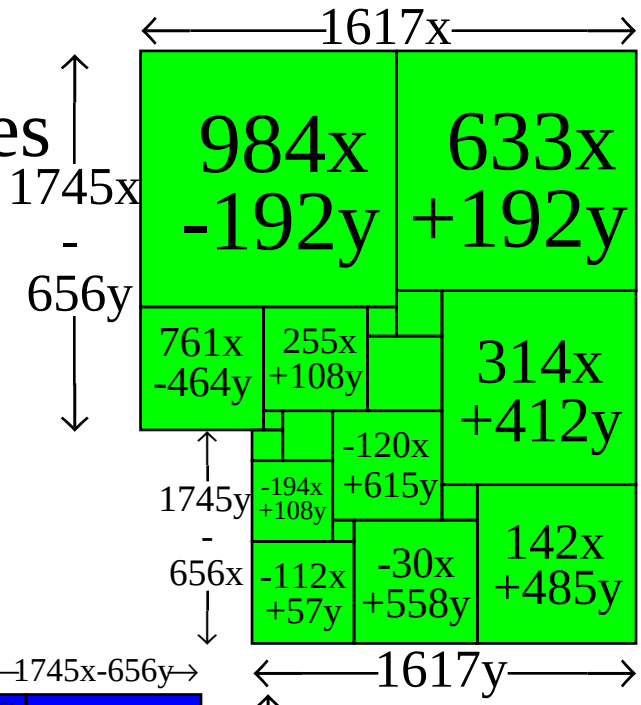



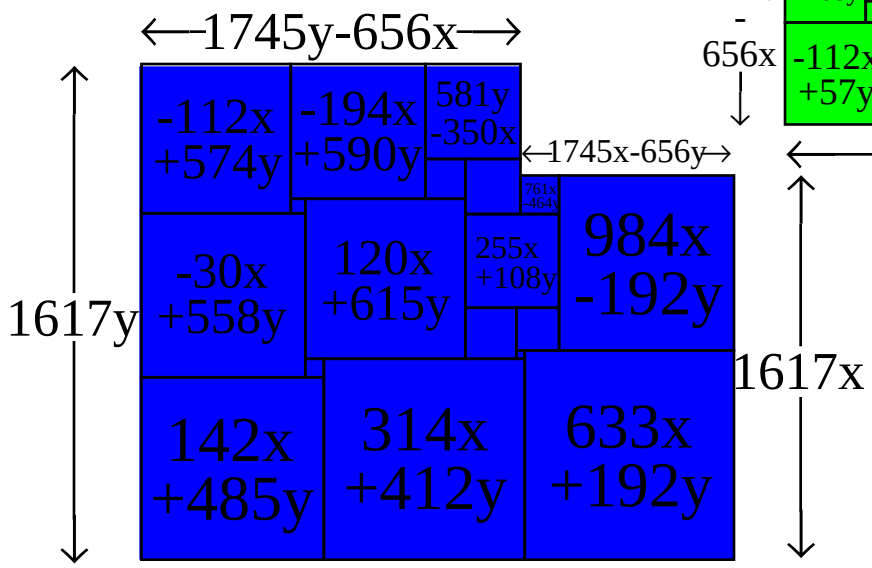

CISS 20:78

# BROOKS'S SPSS




Congruent but different instances of a hexagon externally equivalent to its mirror-image.

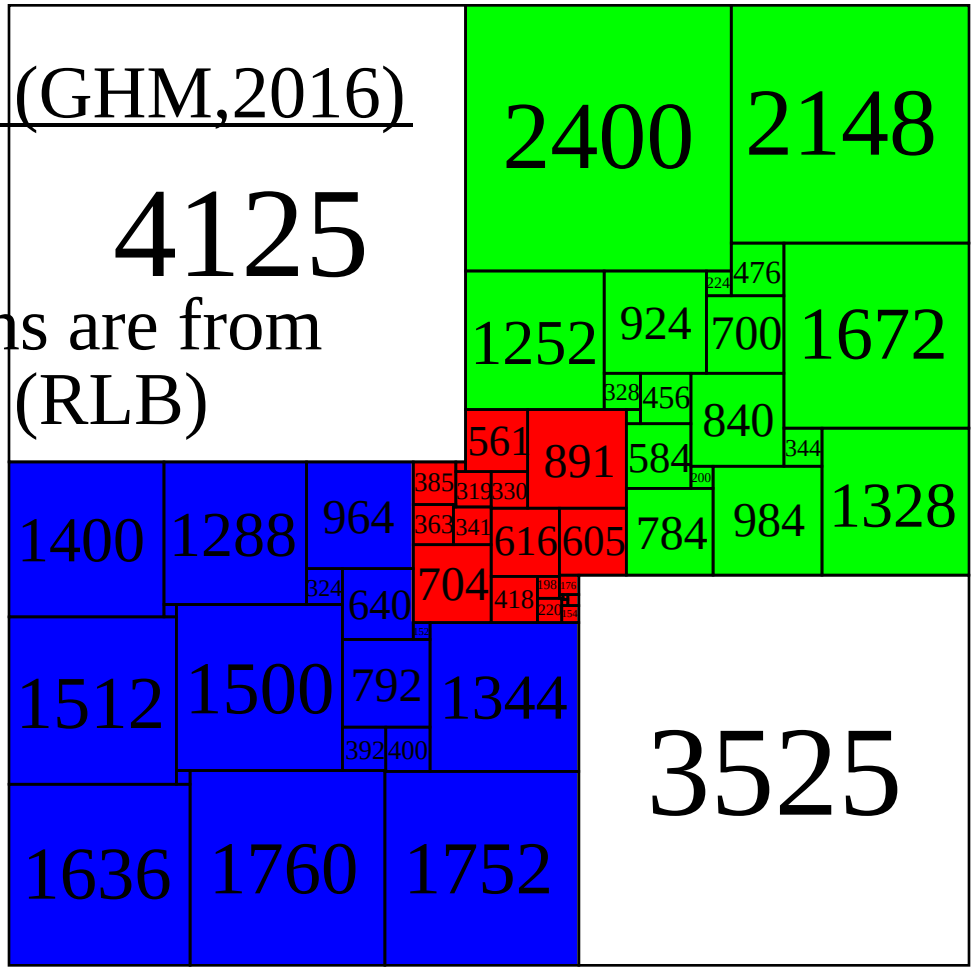


# SECOND GENERALISATION (CONT.)

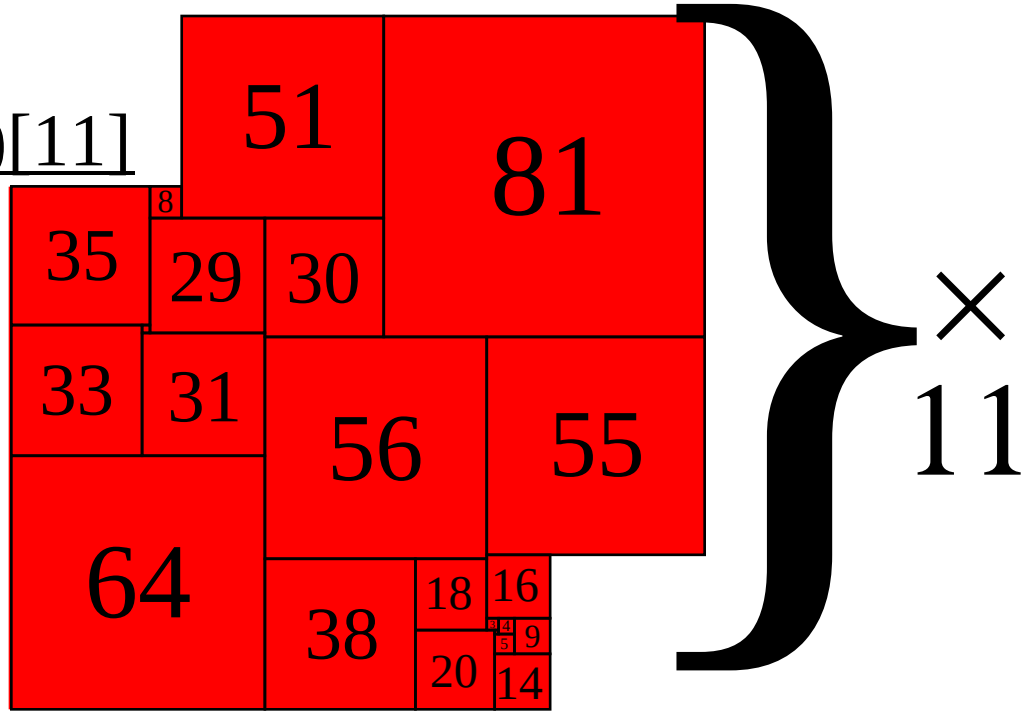
Smallest blemish-free example. From squares mentioned in Tutte's 1950 article.

60:8673 (GHM,2016)




Hexagons are from 38:4920 (RLB)

24:175 (THW,1948)[11]



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[1] C.J. Bouwkamp, On the construction of simple perfect squared squares, Koninkl. Nederl. Akad. Wetensch. Proc. Ser. A 50 (1947) 1296-1299 = Nederl. Akad. Wetensch. Indagationes Math. 9 (1947) 622-625.

[2] C.J. Bouwkamp and A.J.W. Duijvestijn, Album of Perfect Squared Squares of order 26, EUT Report 94-WSK-02, Eindhoven University of Technology, The Netherlands (Dec 1994) xxiii+441 pp. http://alexandria.tue.nl/repository/books/430534.pdf

[3] R.L. Brooks, C.A.B. Smith, A.H. Stone and W.T. Tutte, The dissection of rectangles into squares, Duke Math. J. 7 (1940) 312-340.

[4] R.L. Brooks, C.A.B. Smith, A.H. Stone and W.T. Tutte, A simple perfect square, Koninkl. Nederl. Akad. Wetensch. Proc. Ser. A 50 (1947) 1300-1301.

[5] A.J.W. Duijvestijn, Electronic computation of squared rectangles. Dissertation, Technische Hogeschool, Eindhoven, The Netherlands, 1962; also in Philips Res. Rep. 17 (1962) 523-612.

[6] P.J. Federico, Squaring rectangles and squares: A historical review with annotated bibliography, in: "Graph Theory and related Topics," (J.A. Bondy, U.S.R. Murty, eds.) Academic Press, New York, 1979, pp. 173-196.

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[7] G.H. Morley, Networks and squared squares, Eureka (J. Archimedeans, Cambridge Univ. Math. Soc.) No. 30 (October 1967) 14-16.
[Correction: square side is 616457, not 616467.]

[8] Minutes of the 204th Meeting of the Trinity Mathematical Society (Cambridge) (24 Apr 1939). Minute Books, vol. III, p. 248. Announcement by C.A.B. Smith that Tutte had found a perfect squared square with no perfect subrectangle. (D. Singmaster)

[9] W.T. Tutte, Squaring the square, Canad. J. Math. 2 (1950) 197-209.

[10] W.T. Tutte, Squaring the square, Sci. Amer. 199 (Nov 1958), 136-142. Reprinted with addendum and enlarged bibliography in M. Gardner, ed., 2nd Scientific American Book of Mathematical Puzzles and Diversions, pp. 186-209, 250, Simon and Schuster, New York (1961) which was also published as More Mathematical Puzzles and Diversions, Penguin Books (1966).

[11] T.H. Willcocks, Problem 7523 and solution, Fairy Chess Review 7 (1948) 97 (Aug); 106 (Oct).

[12] T.H. Willcocks, Some squared squares and rectangles, J. Combinatorial Theory 3 (1967) 54-56.
[Correction: 37:1947 was found in 1959 (Tutte), not 1947.]

**Stuart Anderson's website squaring.net has a huge amount of information on squared squares and related problems.**