Rational Origami

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oddsandevenings.com

The Unit Square

Folding to get 1/2, 1/4, 1/8 is easy.

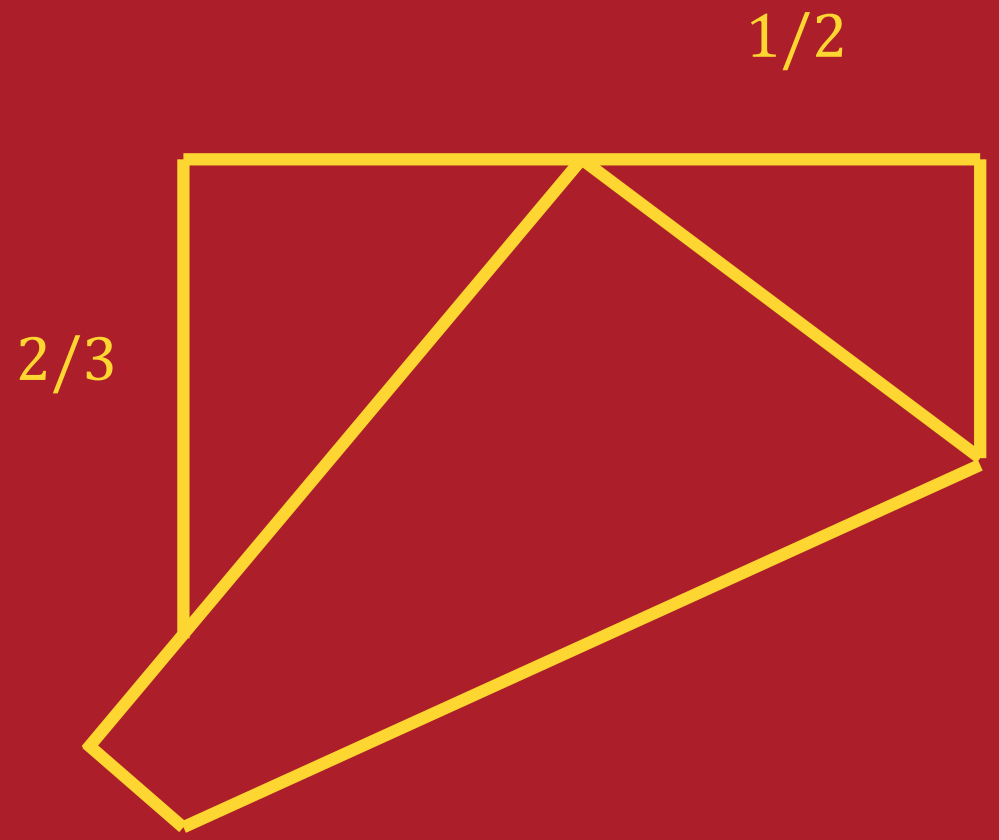
How can we get:

1/3?

1/7?

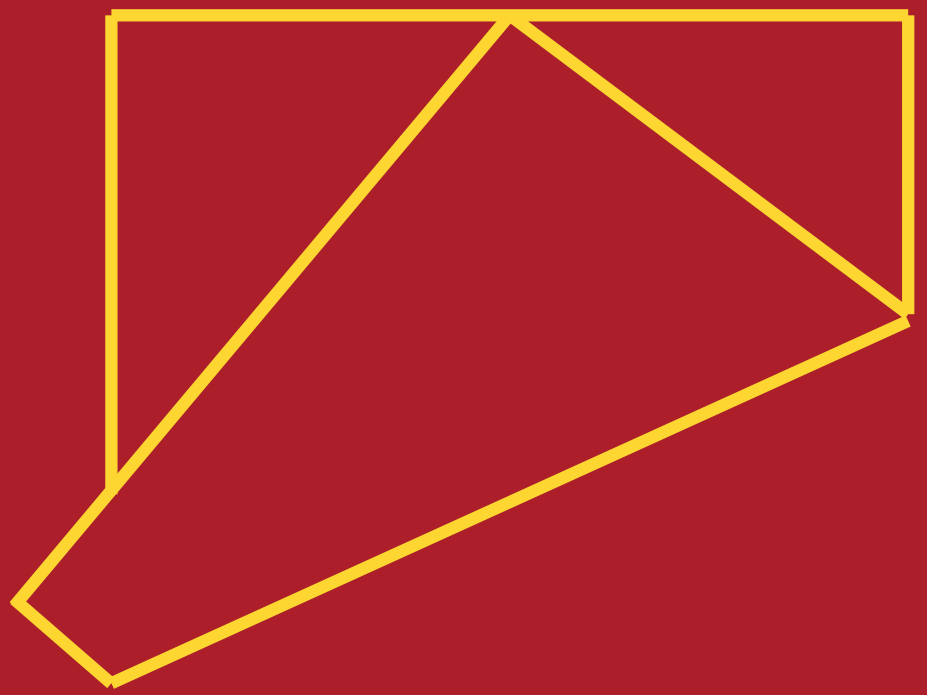
37/55?

The Trick Fold



The Trick Fold

1/2



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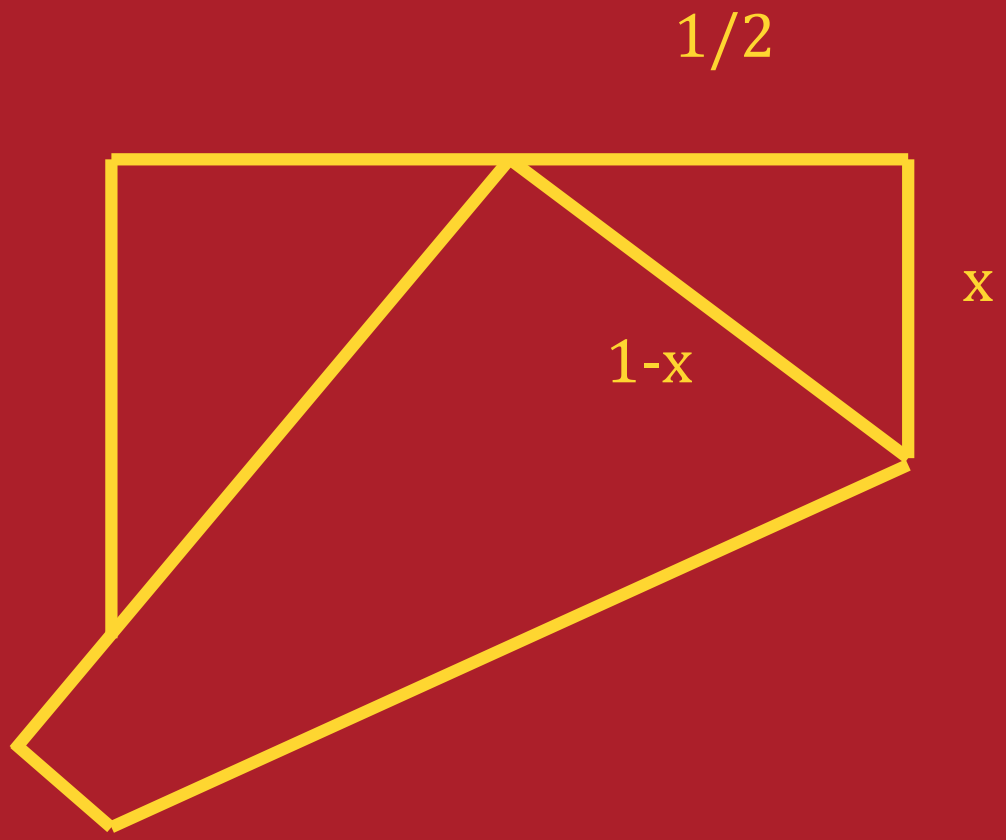

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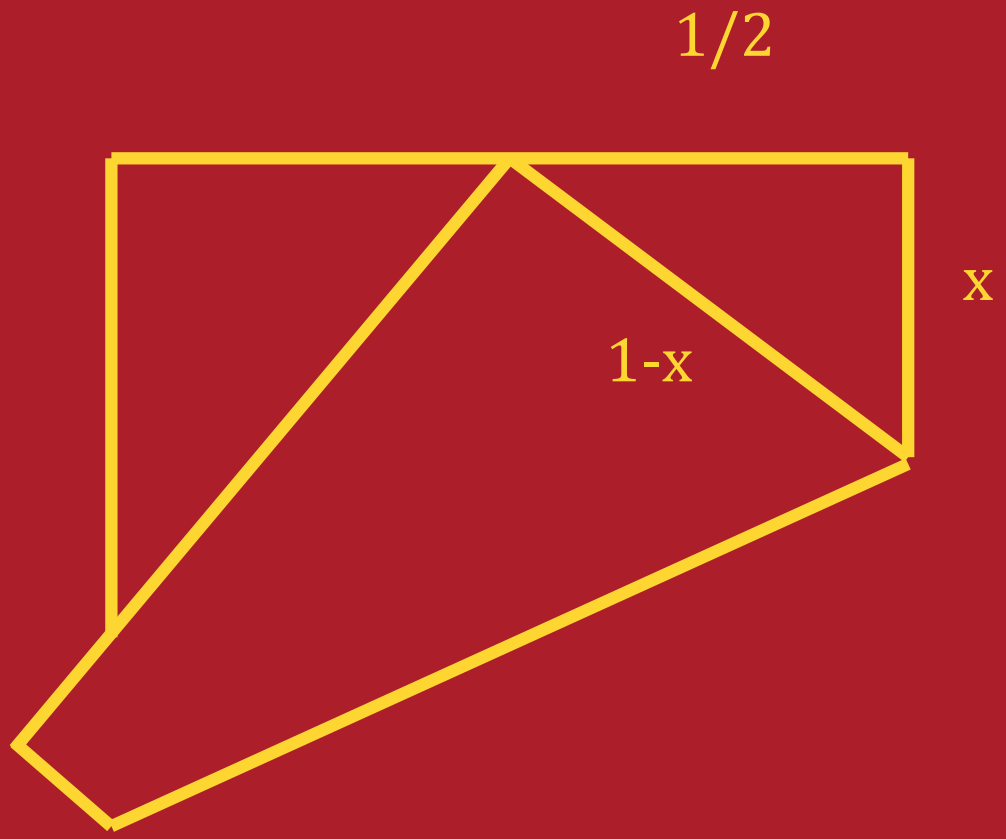
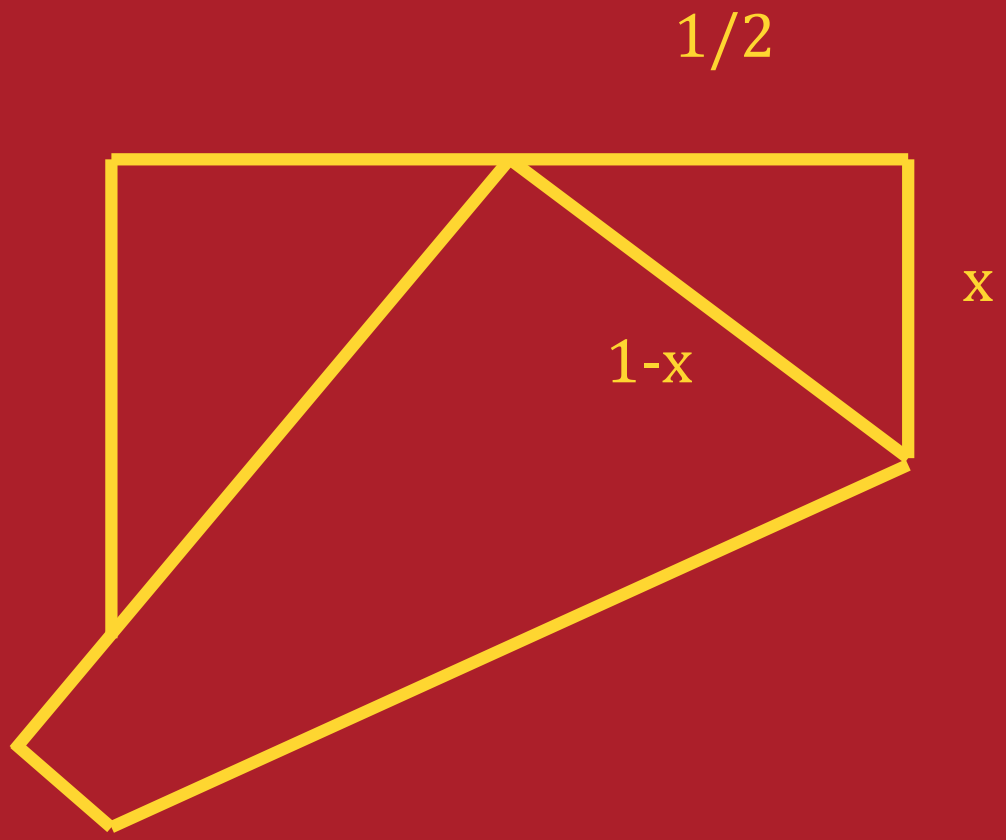


$(1/2)^2+x^2=(1-x)^2$

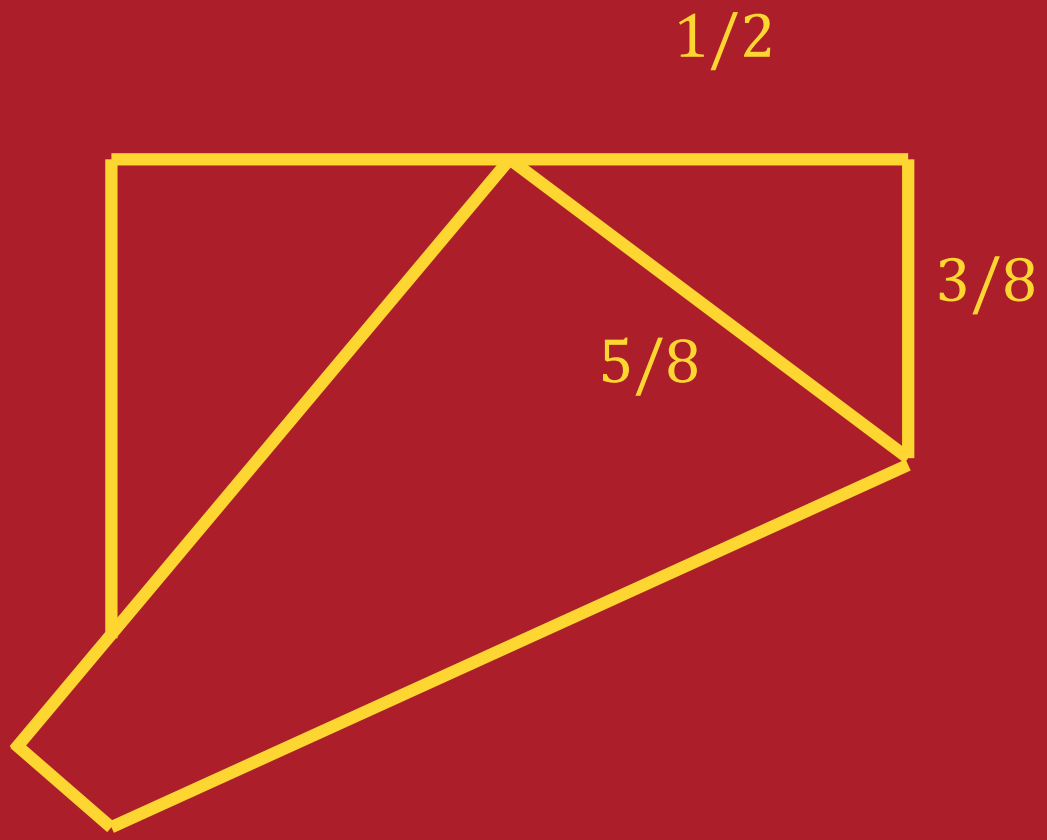
The Trick Fold



$(1/2)^2+x^2=(1-x)^2$

$x=3/8$

The Trick Fold



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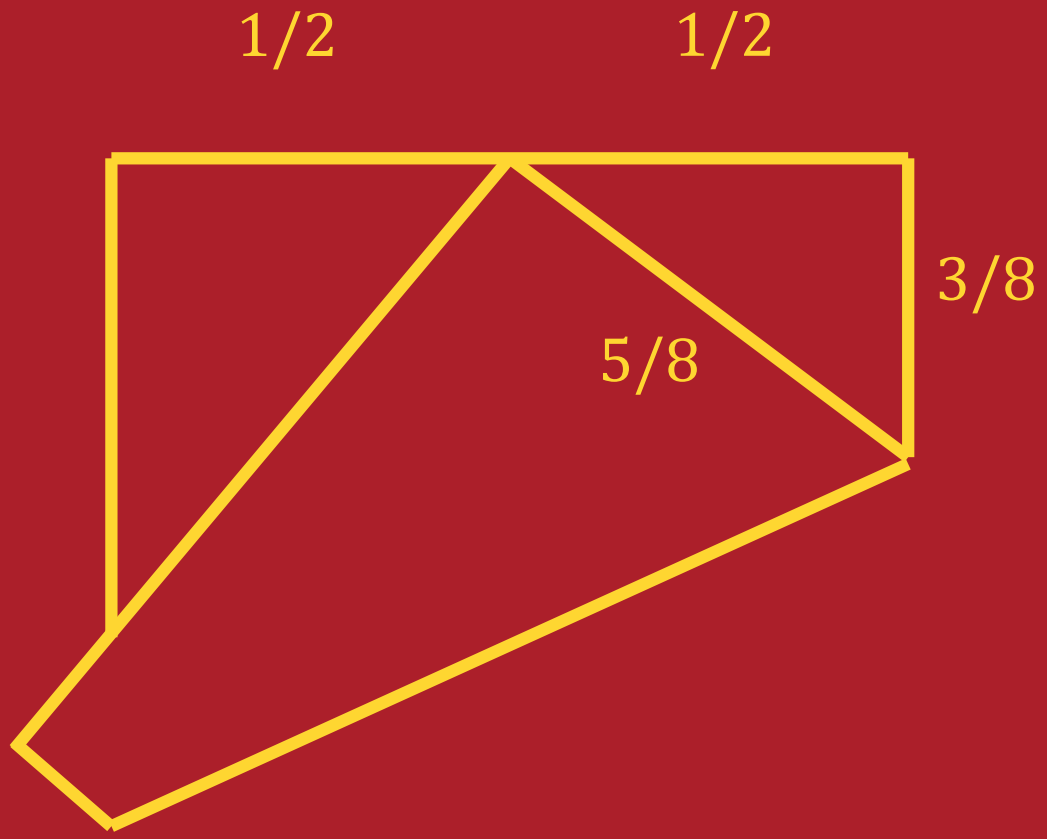

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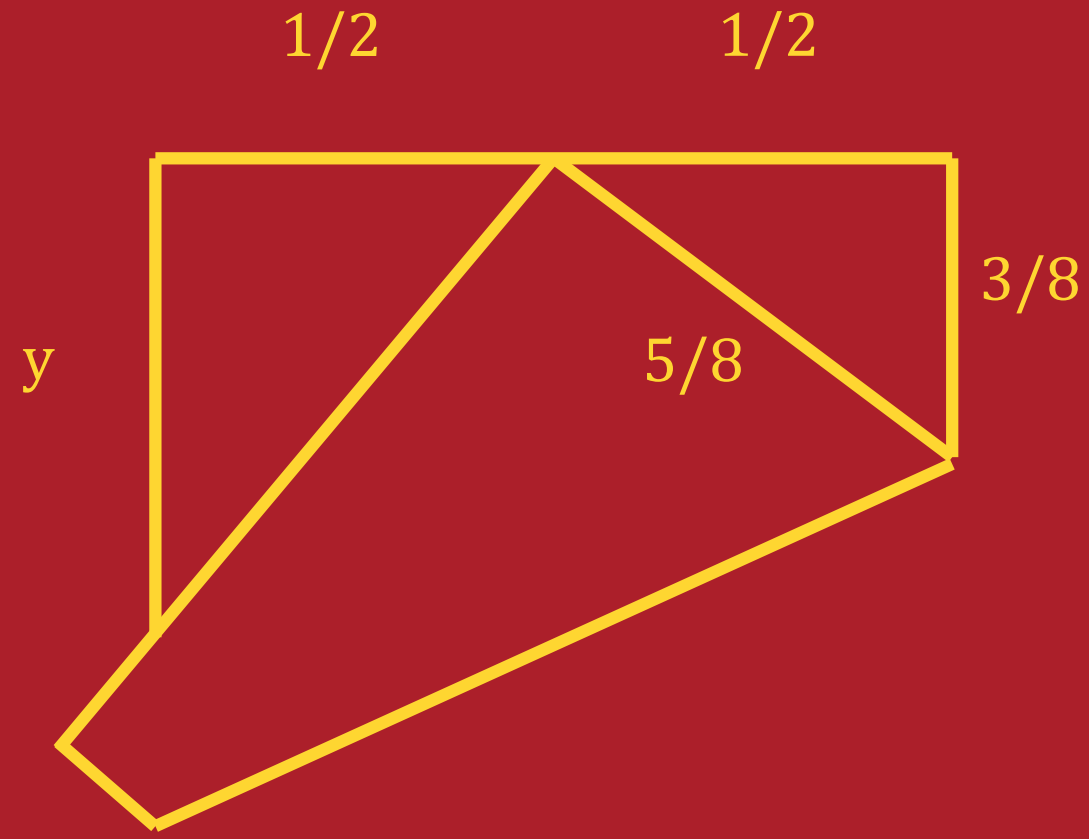
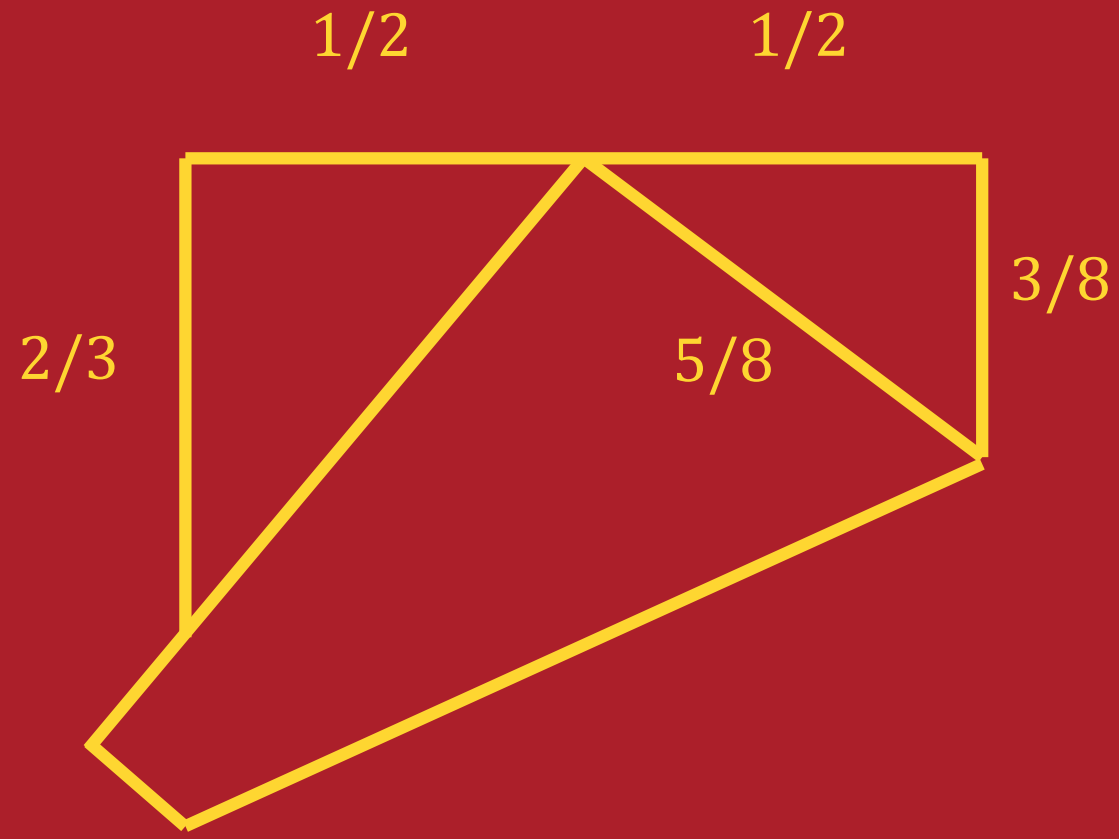


$$\frac{y}{1/2} = \frac{1/2}{3/8}$$

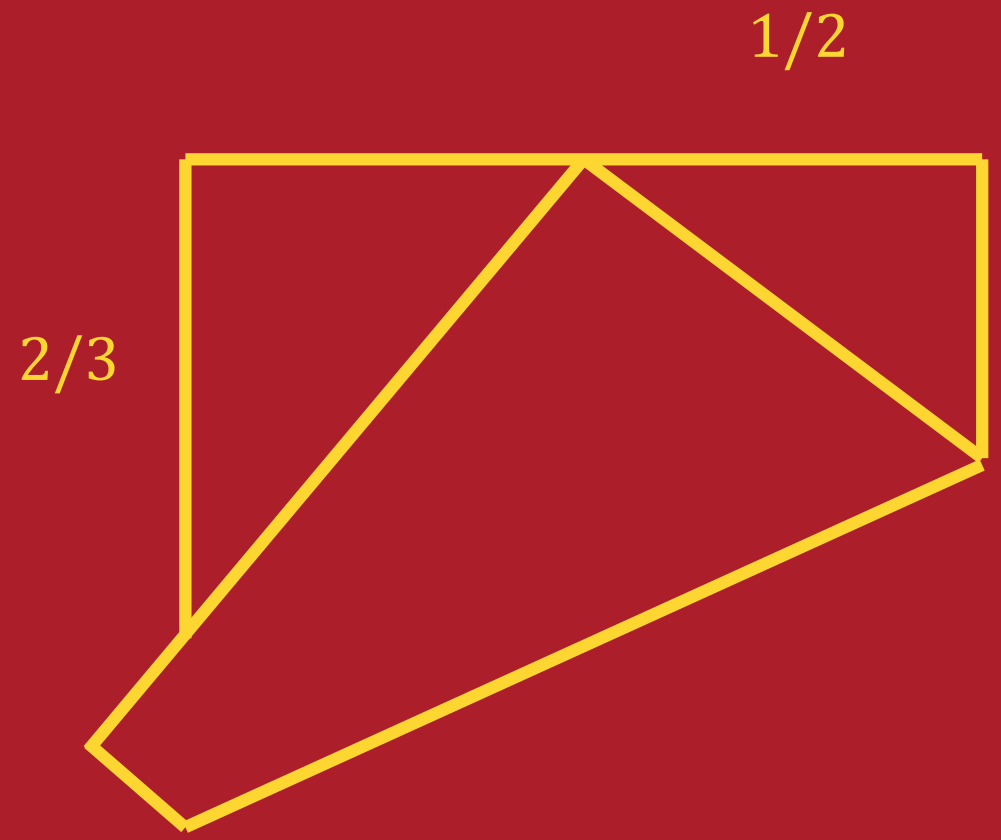
The Trick Fold



$$\frac{y}{1/2} = \frac{1/2}{3/8}$$

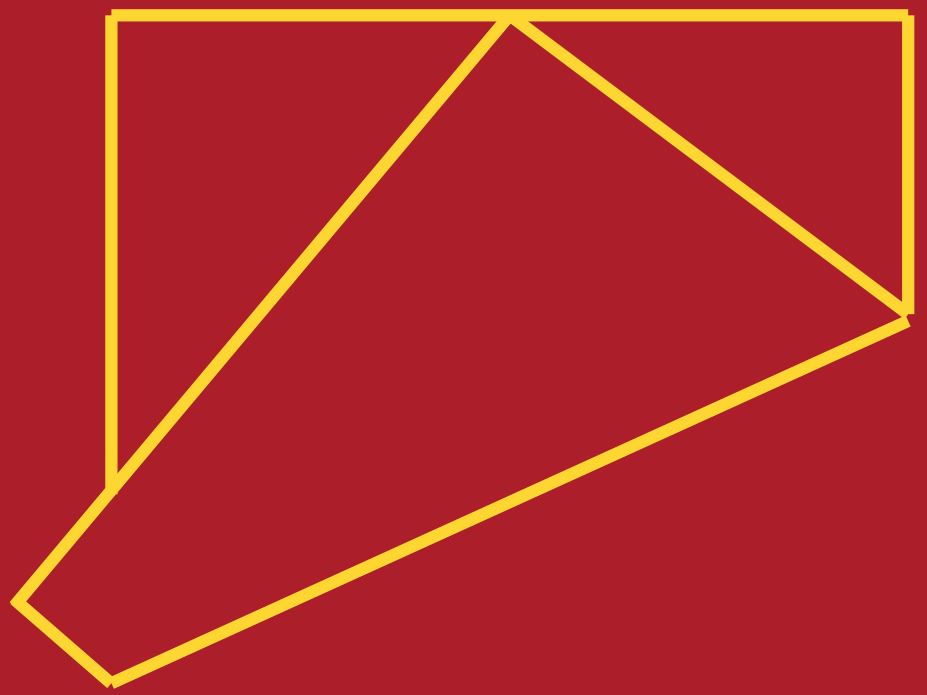
$$y = 2/3$$

The Trick Fold



The Trick Fold

1/n



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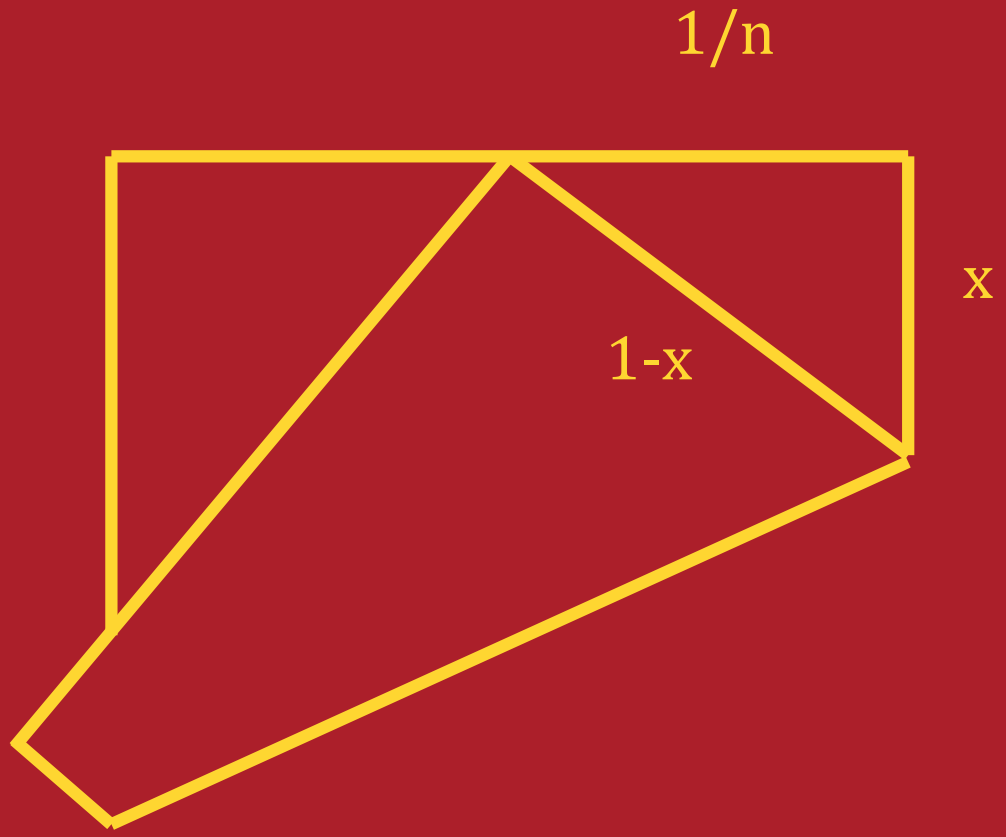

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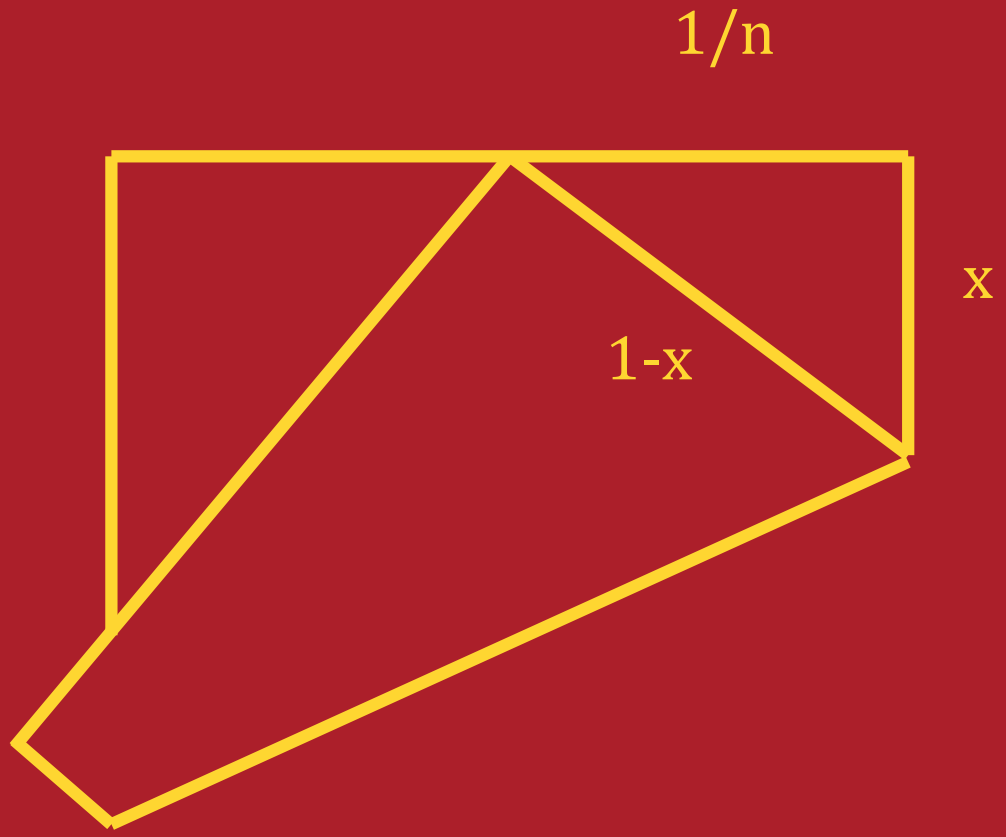
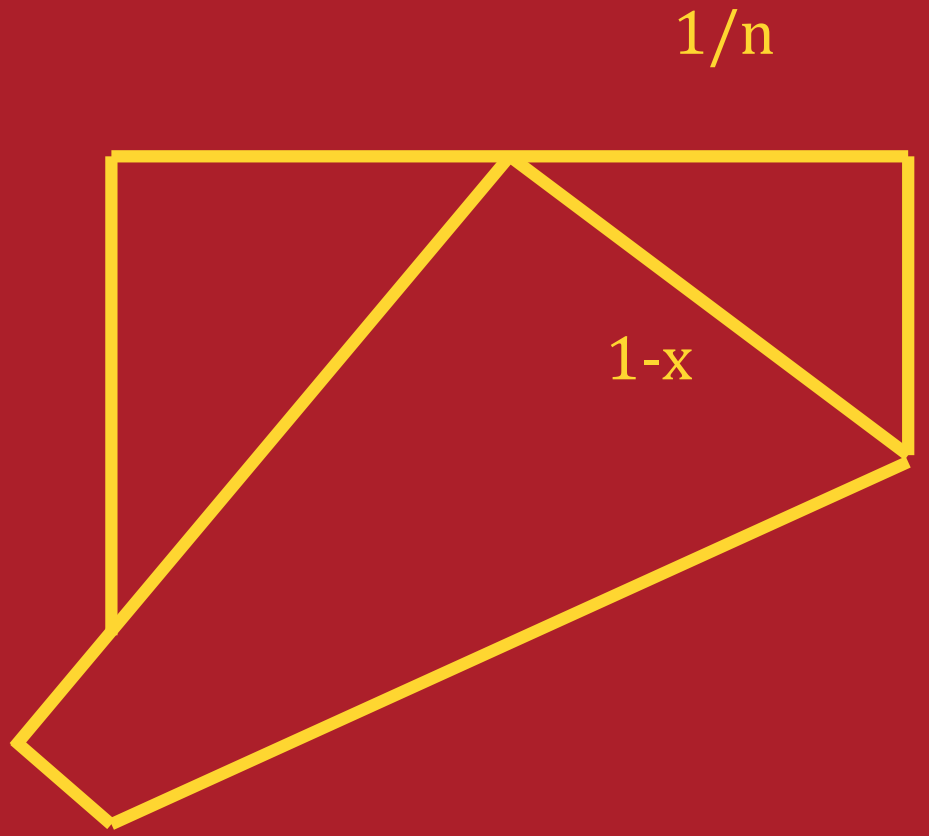


$(1/n)^2+x^2=(1-x)^2$

The Trick Fold

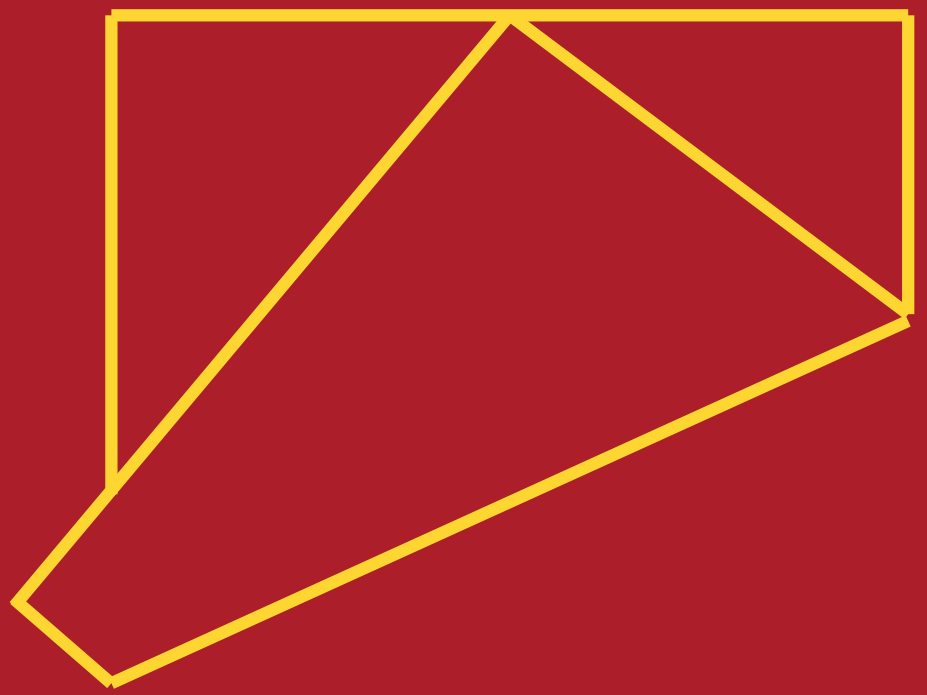


$(1/n)^2+x^2=(1-x)^2$

$x=(n^2-1)/(2n^2)$

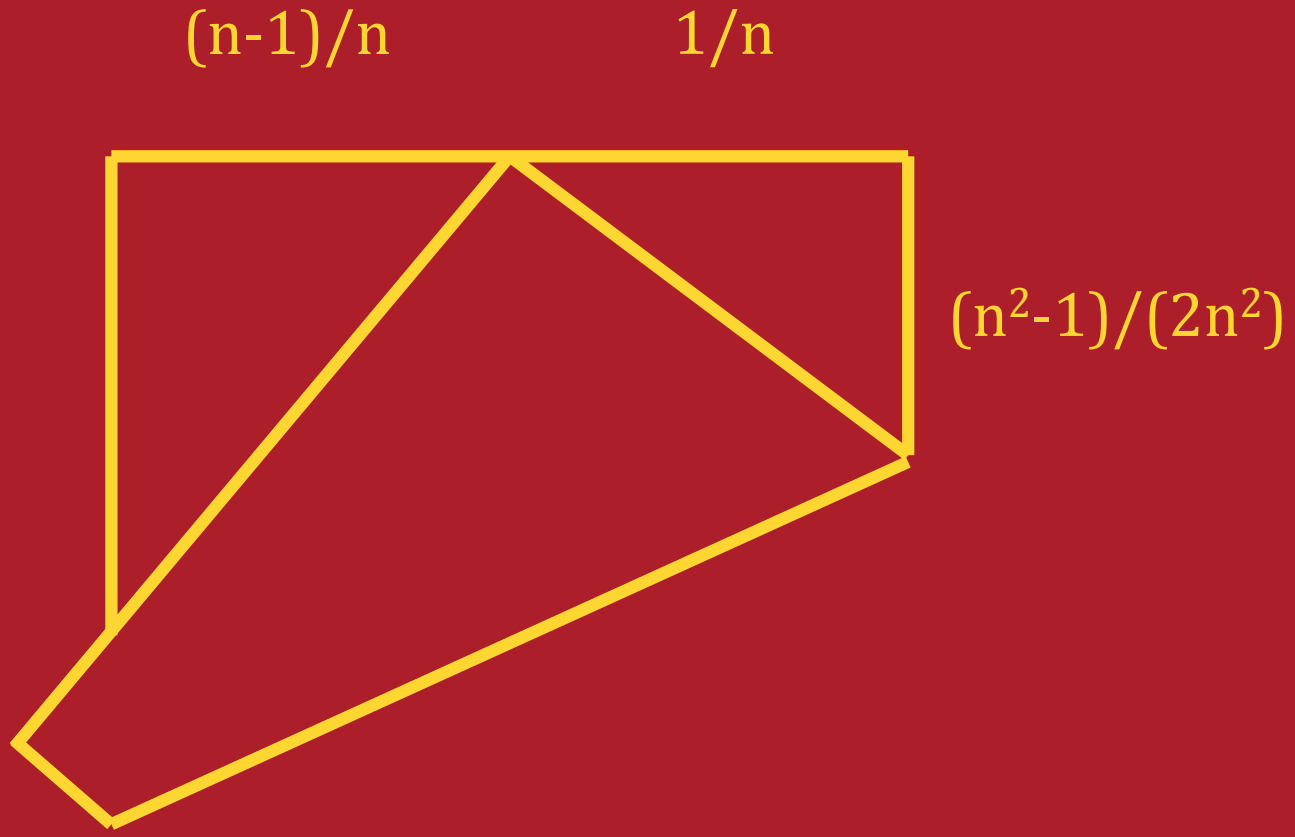
The Trick Fold

$1/n$



$(n^2-1)/(2n^2)$

The Trick Fold



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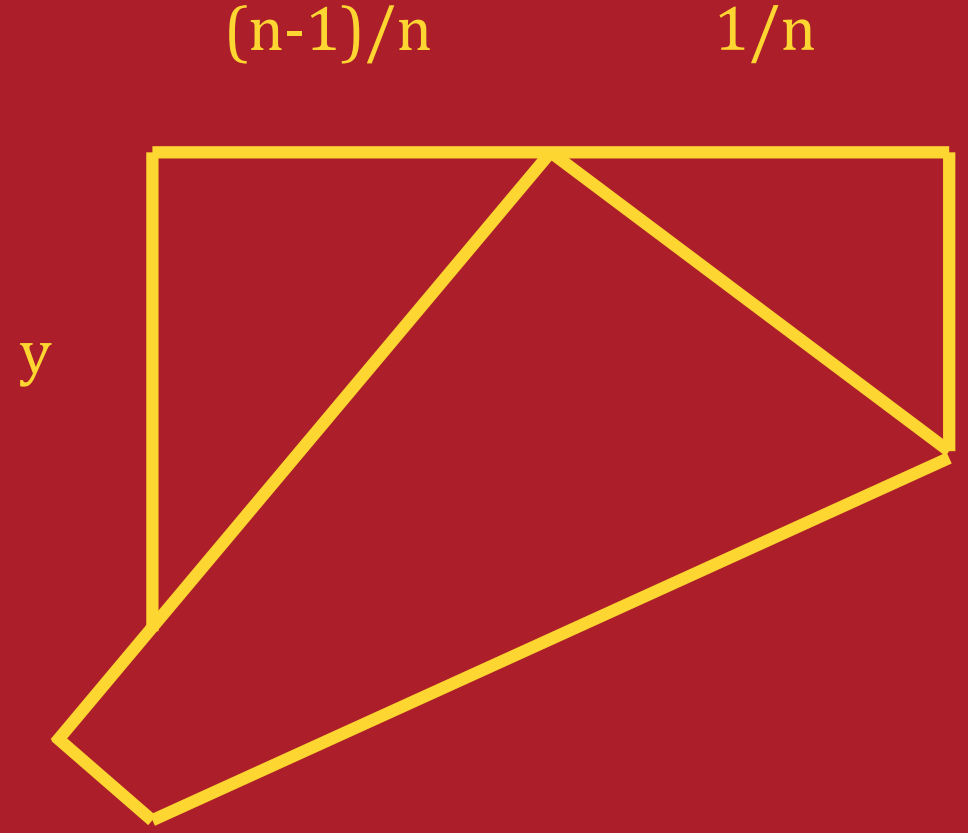
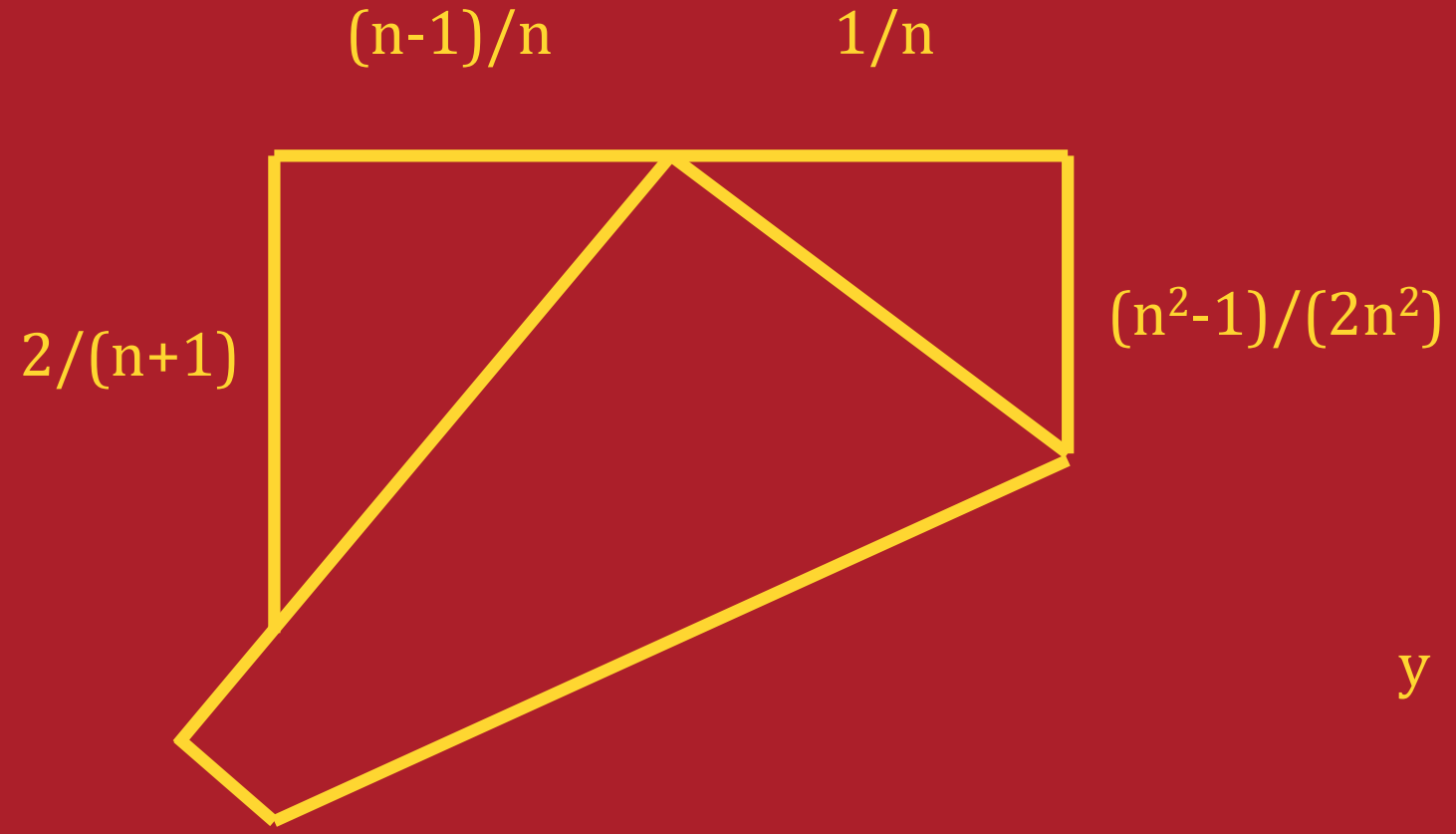


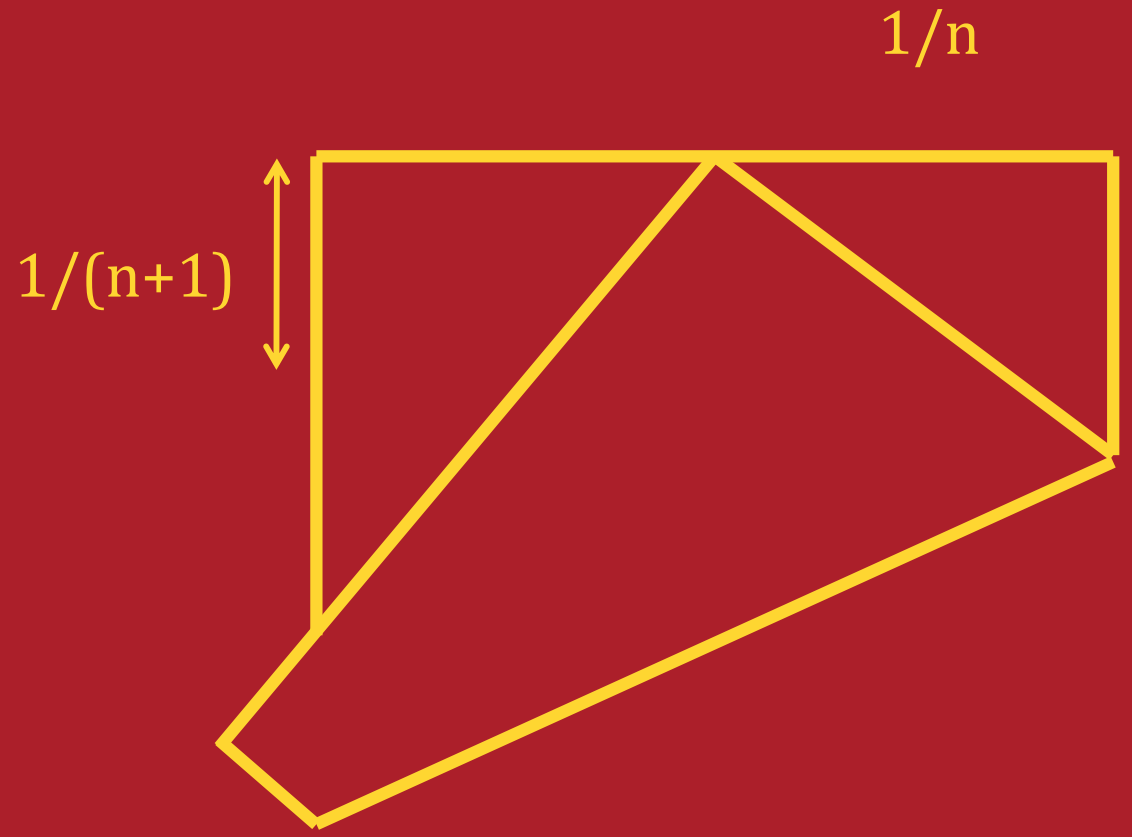
$$\frac{y}{(n-1)/n} = \frac{1/n}{(n^2-1)/(2n^2)}$$

The Trick Fold



$$y = \frac{2(n-1)}{(n^2-1)}$$

The Trick Fold



Our Two Operations

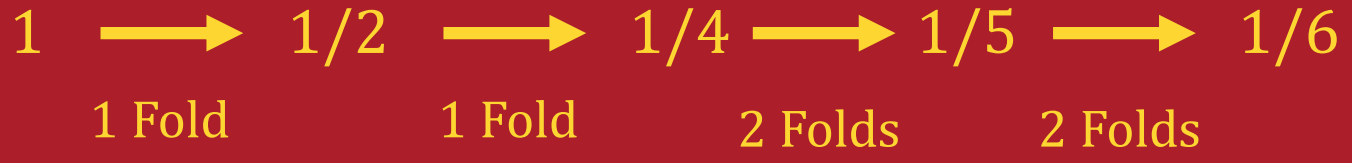
Fold in Half (1 Fold)

$1/n \longrightarrow 1/(2n)$

Our Trick (2 Folds)

$1/n \longrightarrow 1/(n+1)$

Target 1/6



So 6 Folds

Can we do better?

Target 1/6

So 6 Folds

1 → 1/2 → 1/4 → 1/5 → 1/6

1 Fold, 1 Fold, 2 Folds, 2 Folds

1 → 1/2 → 1/3 → 1/6

1 Fold, 2 Folds, 1 Fold

So 4 Folds

The Binary Method

Convert the Target Denominator
to Binary.
Ignore the first digit.

- Every 1 means Fold in Half,
  then Trick Fold
- Every 0 means Fold in Half

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Target
1/54
54 is 1<u>10110</u> in binary

So we take <u>10110</u>

The Binary Method

Convert the Target Denominator to Binary.
Ignore the first digit.

- Every 1 means Fold in Half, then Trick Fold
- Every 0 means Fold in Half

Target
1/54
54 is 1<u>10110</u> in binary

So we take <u>10110</u>