An Unappreciated Result and a Folding Puzzle

Martin Harris, Online MathsJam Conference 2020

The Unappreciated Result

$$P + Q + R = 180°$$

$$P + Q = 180° - R$$

$$\tan(P + Q) = \tan(180° - R)$$

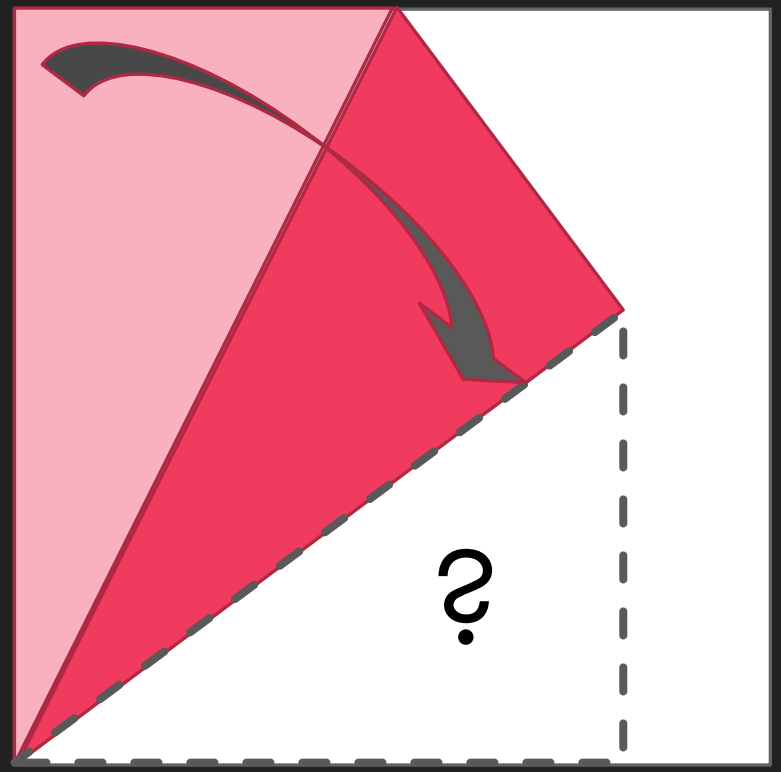
$$\frac{\tan(P) + \tan(Q)}{1 - \tan(P).\tan(Q)} = -\tan(R)$$

$$\tan(P) + \tan(Q) = -\tan(R) + \tan(P).\tan(Q).\tan(R)$$

$$\mathbf{\tan(P) + \tan(Q) + \tan(R) = \tan(P).\tan(Q).\tan(R)}$$

The Folding Puzzle (I)

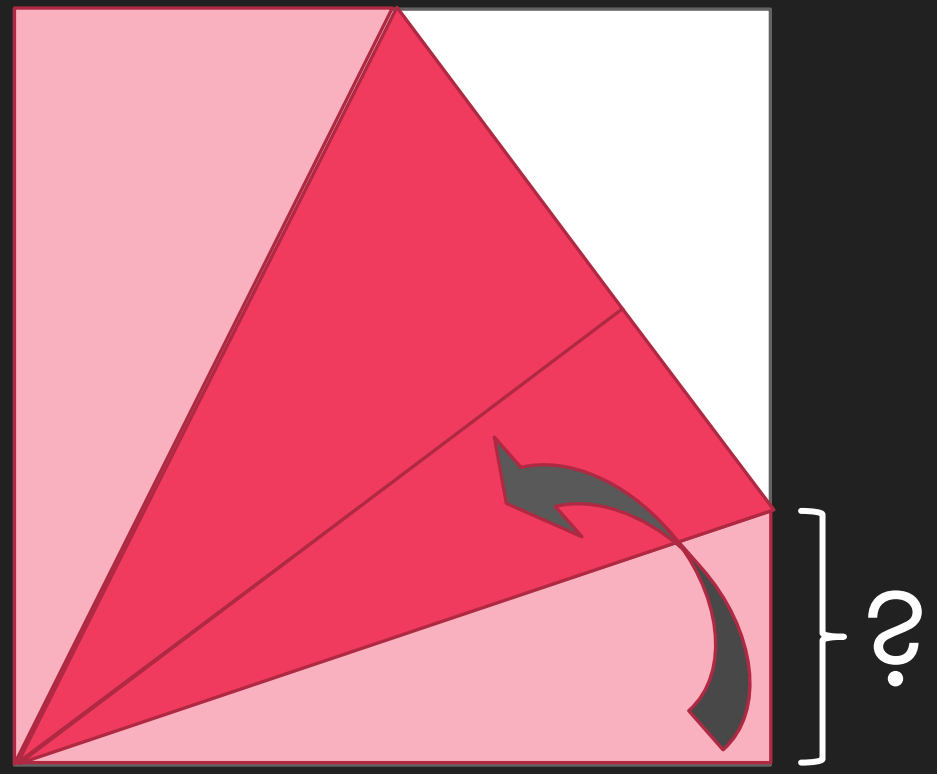
- Take a square of paper, and fold from the midpoint of the top to the bottom left corner. What is the ratio of the sides of the triangle below this line?



Hat tip to Martin Whitworth, who first showed me this on an online MathsJam

The Folding Puzzle (II)

- Now fold the bottom right corner up to meet the other corner. How far up the right hand side is the point where the fold hits?



Back to the Formula

$$\mathbf{\tan(P) + \tan(Q) + \tan(R) = \tan(P).\tan(Q).\tan(R)}$$

IF R = 45°, tan(R) = 1, and the formula can be rearranged to express tan(Q) in terms of tan(P):

$$\tan(P) + \tan(Q) + 1 = \tan(P).\tan(Q)$$

$$\tan(P) + 1 = [\tan(P) - 1] .\tan(Q)$$

$$\mathbf{\tan(Q) = \frac{\tan(P) + 1}{\tan(P) - 1}}$$

Other Examples

End

@MarHarStar

tan(Q) for your time :)