

The Sleeping Beauty Problem

Tony Mann

The Sleeping Beauty Problem

- Puzzle due to Arnold Zunoff (1980s), first published by Adam Elga (2000)
- See Peter Winkler, *Mathematical Puzzles*, and https://en.wikipedia.org/wiki/Sleeping_Beauty_problem

The Problem

- The subject (S) of the experiment goes to sleep on Sunday and sleeps until Wednesday.
- A fair coin is tossed.
- If the coin is heads, S will be woken up once on Monday only.
- If the coin is tails, S will be woken up once on Monday and once on Tuesday.
- S will not remember any previous awakenings. When woken on Monday or Tuesday, they are not told what day it is.

The question

- S knows the full protocol.
- When S is woken during the experiment, what should they think is the probability that the coin came up heads?
- That is, if they had to bet on the coin having been heads, what odds would they think fair?

Argument 1

There are four possible events:

- It is Monday, the coin was heads, and S is awakened
- It is Monday, the coin was tails, and S is awakened
- It is Tuesday, the coin was heads, and S is not awakened
- It is Tuesday, the coin was tails, and S is awakened

So if S is awakened, by counting the equally likely possible cases that remain, the probability coin was heads is $1/3$.

Argument 2

- When S went to sleep on Sunday, they knew probability coin would be heads was $\frac{1}{2}$.
- When they are awakened, they have no new information (they knew they were going to be awakened at some point).
- So when they are awakened S should still think the probability of heads is $\frac{1}{2}$.

Are you a Thirder or a Halfer?

- Over 100 papers published in philosophy journals on this problem

A note

If you are a thirder:

- On Sunday S estimates the probability of heads was $\frac{1}{2}$.
- On Monday and Tuesday, when awake, S thinks it's $\frac{1}{3}$.
- On Wednesday they thinks it's $\frac{1}{2}$ again.
- S knows in advance that they will hold these contradictory beliefs in the future.
- Is this a problem?

Conclusion

Such puzzles are difficult!

Thank you