Is 2023 a Prime Number ?

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R.I.P.

Mathematical Popularisers *Extraordinaires*

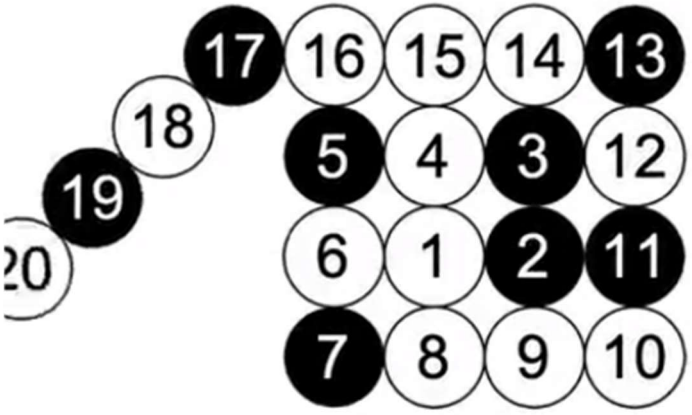
John Sharp	***sliceforms***
Pat Ashforth	***woollythoughts***
Tom Apostol	***Born 20 August 1923***

Is 2023 a prime number? (Discuss)

Number of factors

Visualisations: number line, Ulam spiral (video):

What do you see? (Discuss)



Primes and semiprimes

A rather useless fact left over from last year:
2021 is a special number: It concatenates two *consecutive* integers (20 and 21)
and is also the product of two *consecutive* primes (43 and 47).
The next such number has 32 digits: **2307340946901148 2307340946901147**

So - 2021 is a *semiprime* – the product of two primes

2022 and 2023 are neither prime nor semiprime.

Number of Factors: (Legendre?) Theorem

Divisibility tests: 2 3 4 5 6 8 9 11 'easy'. 7 13 17: less easy, but

If number is $x = 10a+b$, then:

x div 7 iff $a+5b$ div 7 2023 -> 202+15 -> 217 *so* div 7

x div 17 iff $a-5b$ div 17 2023 -> 202-15 -> 187 *so* div 17.

Primes around 2023

Nearest primes are 2017, 2027.

14 prime numbers between 2000 and 2100 (14%):

2003 2011 17 27 29 39 53 63 69 81 83 87 89 99

These include three sets of prime twins.

Two triplets sets start at 2081 and 2083.

These are the only triplets between 1997 and 2137.

Density of primes, and Prime differences (gaps)

Avg difference =¬ 1/ log n

How many primes this century 2000-2099?

We expect 1/ log 2050 = 13.1%

In fact there are 14.

Special Numbers, Special Primes

Twin primes p p+2

Prime triplets: p p+2 p+6 or p p+4 p+6

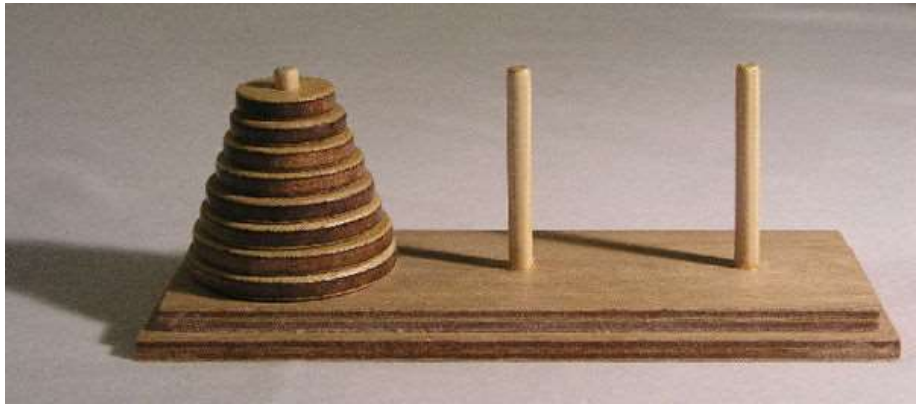
Mersenne Numbers 3 7 15 31 63 = 2^n-1

Number of moves in Benares problem

Mersenne Primes 3 7 31 127 8191

In all, 51 Mersenne primes are known. The largest is $2^{82,589,933}$ -1 (24 million digits).

It is the largest known prime number.



Germain Primes **p 2p+1 both prime.** The only Germain primes this century are 2003 2039 2063 2069.

Germain (Cunningham) chains. 2 starts a 5-member Germain (Cunningham) chain: 2 5 11 23 47

2023 is not prime. Its closest primes are 2017 and 2027.
However neither of these is a Germain prime.
(In fact, no number ending in 7 is a Germain prime. Why?)

Why are primes special? Multiplicative building blocks

Prime parallels

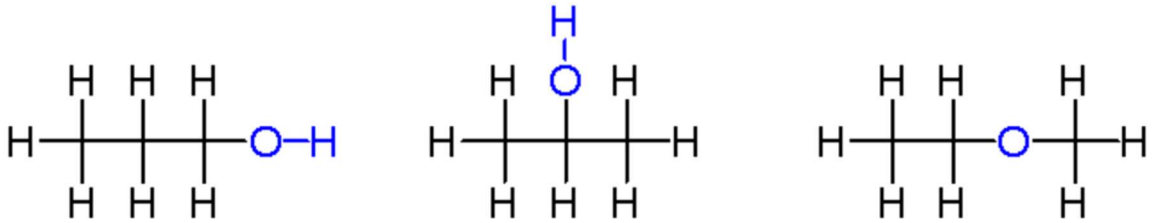
Knots

Colours

Music

Chemistry:

UNLIKE PRIME NUMBERS, chemistry has isomers
e.g. three distinct compounds with the formula C_3H_8O



Uses of Prime Numbers: Cryptography

A detour via Harshad (Niven) numbers:

Numbers that divide by their digital sum

H_7: *n* divides by 7 *and* digisum(*n*) = 7 e.g. 2023

Consider

H_7^+: *n* divides by 7 *and* digiroot(*n*) = 7

H_7^+ bears an uncanny resemblance to OEIS A003363:

Numbers that are the sum of 7 positive 6th powers

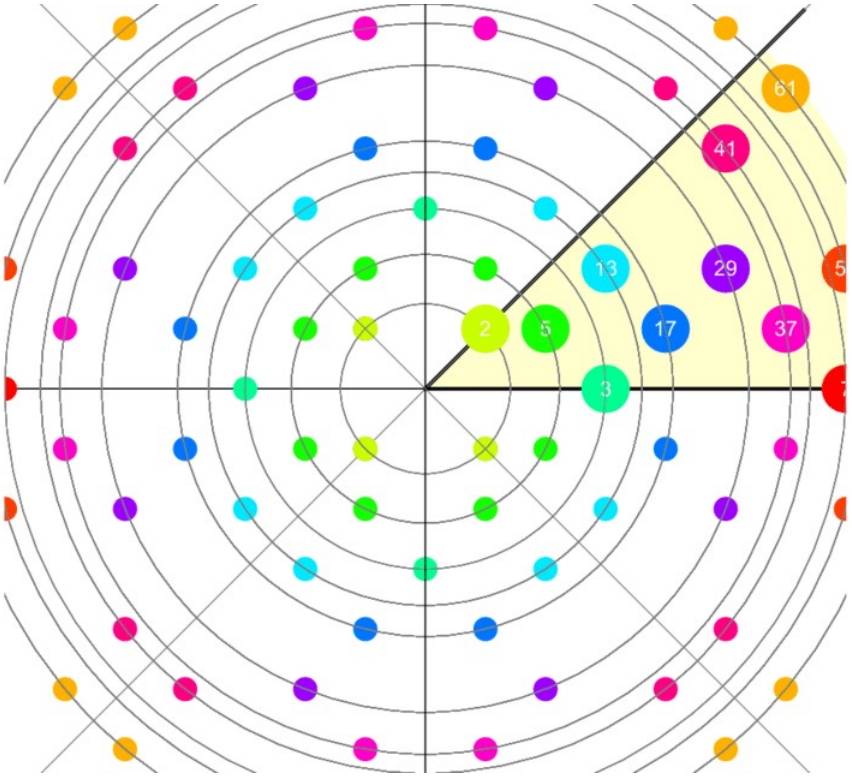
Why ?

Parting Questions:

1. Gaussian Primes

Coordinates

x	y
0	3 7 11
1	1 2 4 6 10
2	1 3 5 7
3	0 2 8 10
4	1 5 9
5	2 4 6 8
6	1 5
7	2 8 10
9	4 10
10	1 3 7 9

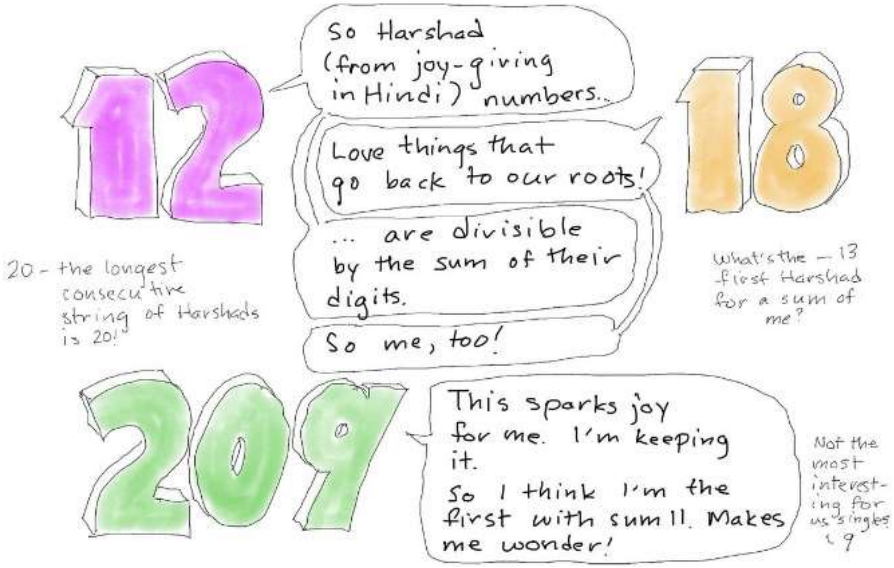


2. Goldbach pairs: 2023 = $p_1 + p_2$; Which pair minimises $/p_1 - p_2/$?

3. Ulam spiral: Where are visuals showing semiprimes, number of factors, Harshad numbers, etc. … ?

4. What number is next in the sequence

511 1015 2023 …. ?



<u>Clue</u>: Harshad = "Joy giving" in Sanskrit

<u>I hope this has given you a little bit of joy</u>

Thank you !!

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