Inequalities and inflation measures

Justifying the means

Alistair Bird - MathsJam Conference 2022

Cost of everything

RPI: Retail price index

1956

2003

CPI: Consumer price index

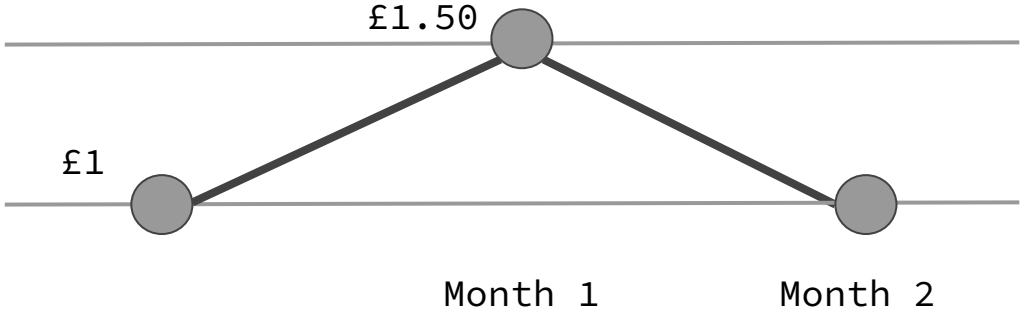
1996

2017

CPIH: CPI with Housing

Lesser of two evils

The price is right



$$\text{Inflation} = \frac{p_{\text{new}}}{p_{\text{old}}}$$

Month 1: $\frac{£1.50}{£1}$ (50%)

Month 2: $\frac{£1}{£1.50}$ (-33.3%)

Overall: $\frac{£1.50}{£1} \times \frac{£1}{£1.50} = 1$

Basket cases

- Both indices are calculated as a weighted average of ~700 representative items (goods and services) with more than 180,000 prices collected.
    - In RPI:

$$I_{\mathrm{A}} := \sum_{i=1}^{n} w_i \frac{p_i^{\mathrm{new}}}{p_i^{\mathrm{old}}}$$

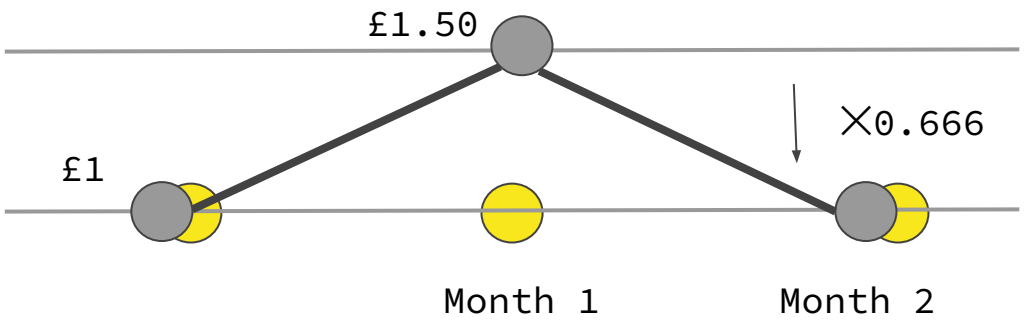
Equally-weighted
Average of relatives
(Carli index)

$$\frac{1}{n}\sum_{i=1}^{n}\frac{p_i^{\mathrm{new}}}{p_i^{\mathrm{old}}}$$

Weighted by price
Ratio of averages
(Dutot index)

$$\frac{\frac{1}{n}\sum_{i=1}^{n} p_i^{\mathrm{new}}}{\frac{1}{n}\sum_{i=1}^{n} p_i^{\mathrm{old}}} = \frac{\sum_{i=1}^{n} p_i^{\mathrm{new}}}{\sum_{i=1}^{n} p_i^{\mathrm{old}}}$$

Shop til it drops



Combining Carli index across months:

½(1.5 + 1) × ½(0.666 + 1)

= 1.25 × 0.888

= 1.10

10% inflation when nothing has changed!

Known as *price bounce*.

To avoid price bounce we want inflation measures with the form:

$$\frac{f(t_2)}{f(t_1)} \times \frac{f(t_3)}{f(t_2)} = \frac{f(t_3)}{f(t_1)}$$

Geometry

Taking the geometric mean gives the *Jevon's index*.

It gives greater weight to lower-priced goods (*substitution*), which is a good feature.

$$\frac{\left(\prod_{i=1}^{n} p_i^{\text{new}}\right)^{\frac{1}{n}}}{\left(\prod_{i=1}^{n} p_i^{\text{old}}\right)^{\frac{1}{n}}} = \left(\prod_{i=1}^{n} \frac{p_i^{\text{new}}}{p_i^{\text{old}}}\right)^{\frac{1}{n}}$$

CPI uses this mean to measure inflation.

Arithmetic-Geometric inequality

Arithmetic mean is always bigger than the geometric mean:

$$\frac{1}{n}(x_1 + x_2 + \cdots + x_n) \geq \sqrt[n]{x_1 \cdot x_2 \cdots x_n}$$

Eg. ½(6+4) = 5 = √25 > √24 = √(6×4).

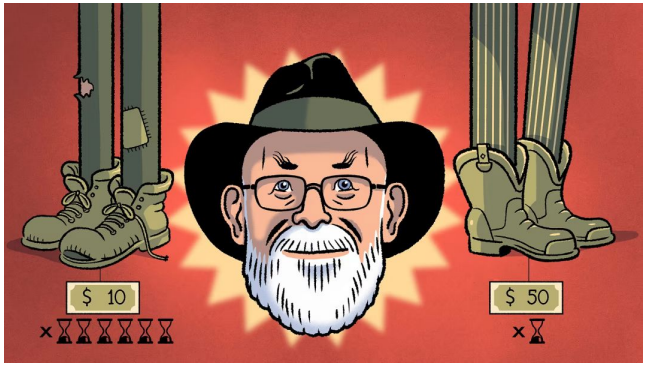
So, because

AM ≥ GM,

then approximately we have

RPI ≳ CPI.

Independent means



- Big Mac Index (The Economist)
  - Global, consistent bundle of goods and wages, easy to collect!
- Vimes Boot Index (Jack Monroe)
  - Low-cost supermarket goods
- Minimum Income Standard (Joseph Rowntree Foundation)
  - Basket of goods is minimum to participate in society

Index Linked

https://outofthenormmaths.wordpress.com/2011/10/04/an-inequality-for-the-consumer-and-retail-price-indices/

GOODBUY!