

The Laplace Transform

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The Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_{t=0}^{\infty} f(t)e^{-st} dt$$

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Transforms a function of time



The Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_{t=0}^{\infty} f(t)e^{-st} dt$$

Transforms a function of time

into a function of s

What on earth is s ?

- Complex frequency
 - Imaginary part **is** frequency
 - Real part is exponential growth/decay

Laplace > Fourier

- The Fourier transform is a special case of the Laplace transform

Laplace

$$s = \sigma + i\omega$$

Fourier

$$s = i\omega$$

Laplace > Fourier

- The Fourier transform is a special case of the Laplace transform

Laplace

$$s = \sigma + i\omega$$



Complex frequency

Fourier

$$s = i\omega$$



Frequency

Differential Equations

- The Laplace Transform turns calculus into algebra

$$\mathcal{L}\{f'\} = s\mathcal{L}\{f\} - f(0)$$

$$\mathcal{L}\{f''\} = s^2\mathcal{L}\{f\} - sf(0) - f'(0)$$

Differential Equations

$$y'' - y' - 2y = 0 \quad y(0) = 1, y'(0) = 0$$

$$[s^2 Y(s) - sy(0) - y'(0)] - [sY(s) - y(0)] - 2Y(s) = 0$$

$$s^2 Y(s) - s - sY(s) + 1 - 2Y(s) = 0$$

$$Y(s) = \frac{s - 1}{s^2 - s - 2}$$

Differential Equations

$$Y(s) = \frac{s - 1}{s^2 - s - 2}$$

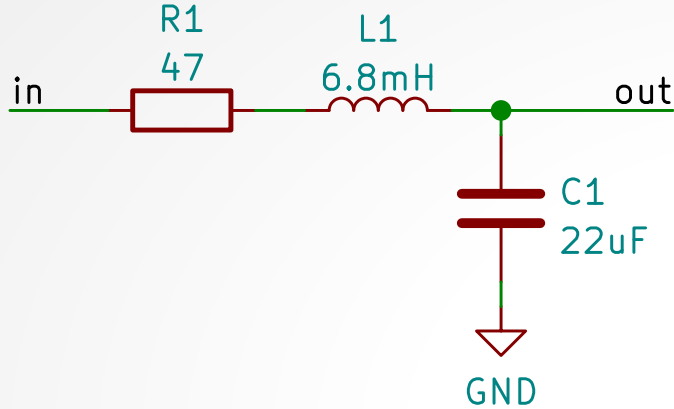
$$Y(s) = \frac{s - 1}{(s - 2)(s + 1)}$$

$$Y(s) = \frac{\frac{1}{3}}{s - 2} + \frac{\frac{2}{3}}{s + 1}$$

$$y(t) = \frac{1}{3}e^{2t} + \frac{2}{3}e^{-t}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s - a}$$

Electronics



$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}}$$

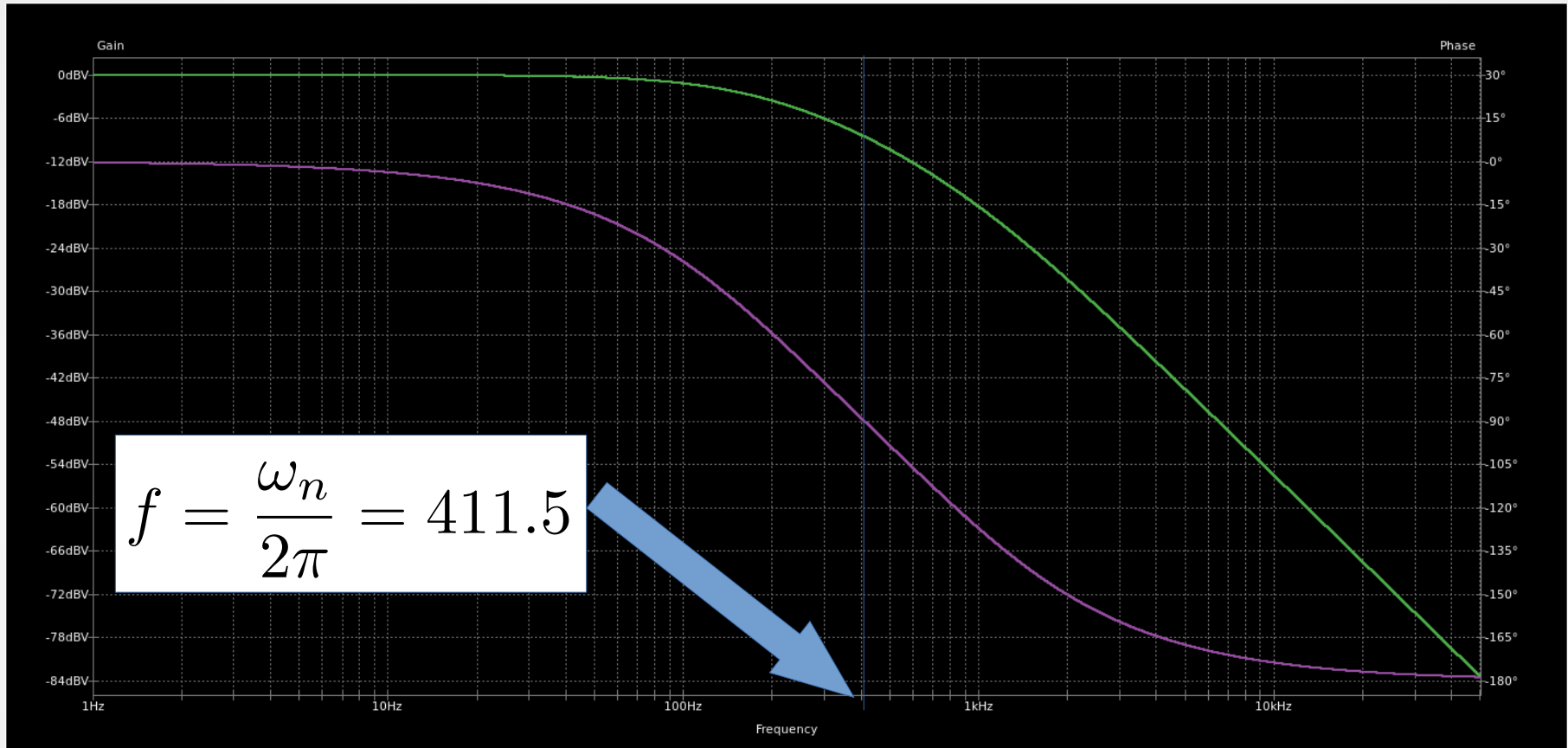
$$\frac{V_{out}}{V_{in}} = \frac{1}{s^2 LC + sRC + 1}$$

$$\frac{V_{out}}{V_{in}} = \omega_n^2 \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

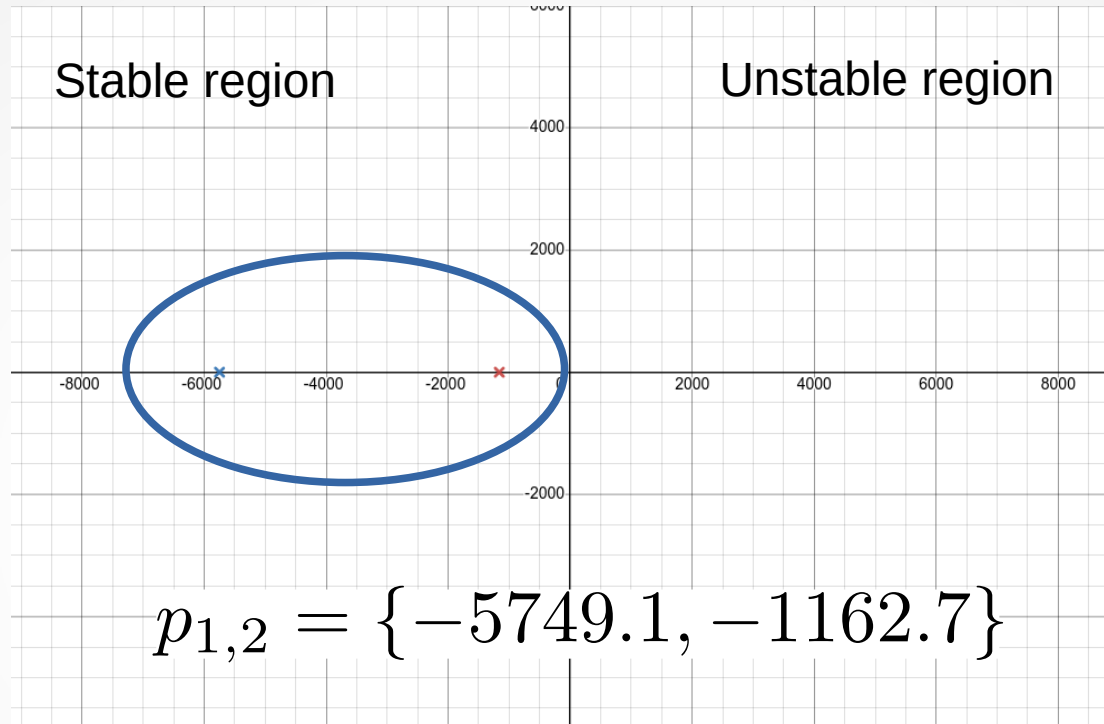
$$\omega_n = \sqrt{\frac{1}{LC}} = 2585.4$$

$$\zeta = \frac{R}{2} \sqrt{\frac{C}{L}} = 1.34$$

Electronics



Pole Zero Plot



All poles are in the stable region, so this system is stable

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