The No-Three-In-A-Line problem

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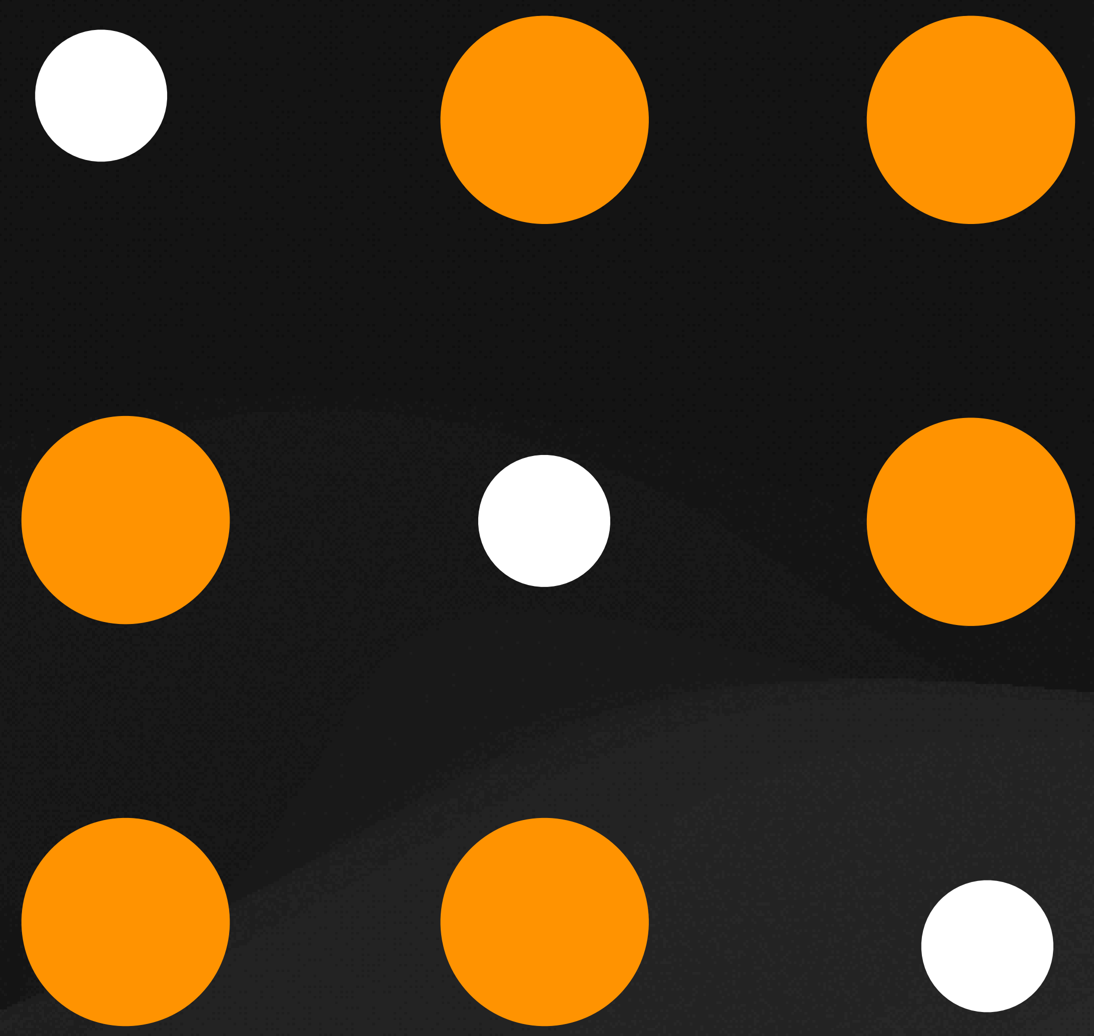
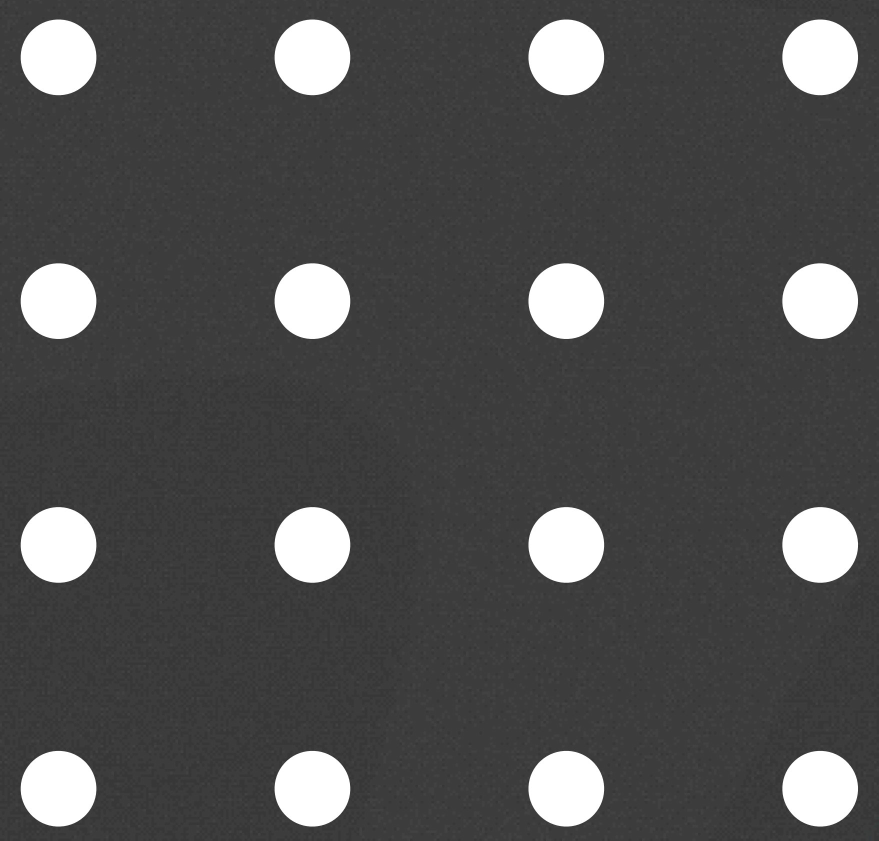
MathsJam 2022



3 by 3 grid

place 6 pieces

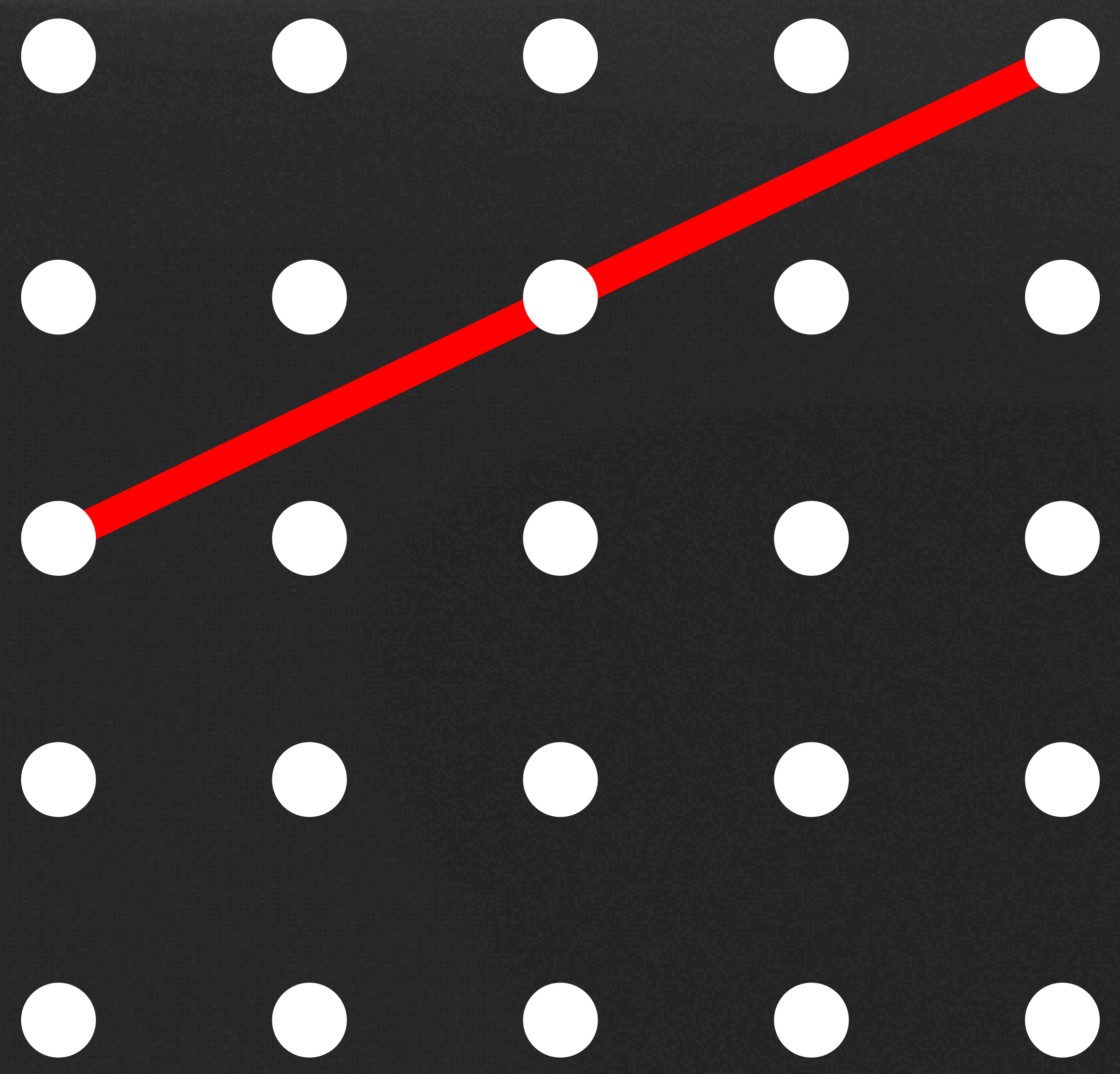
so that no three pieces lie in a straight line

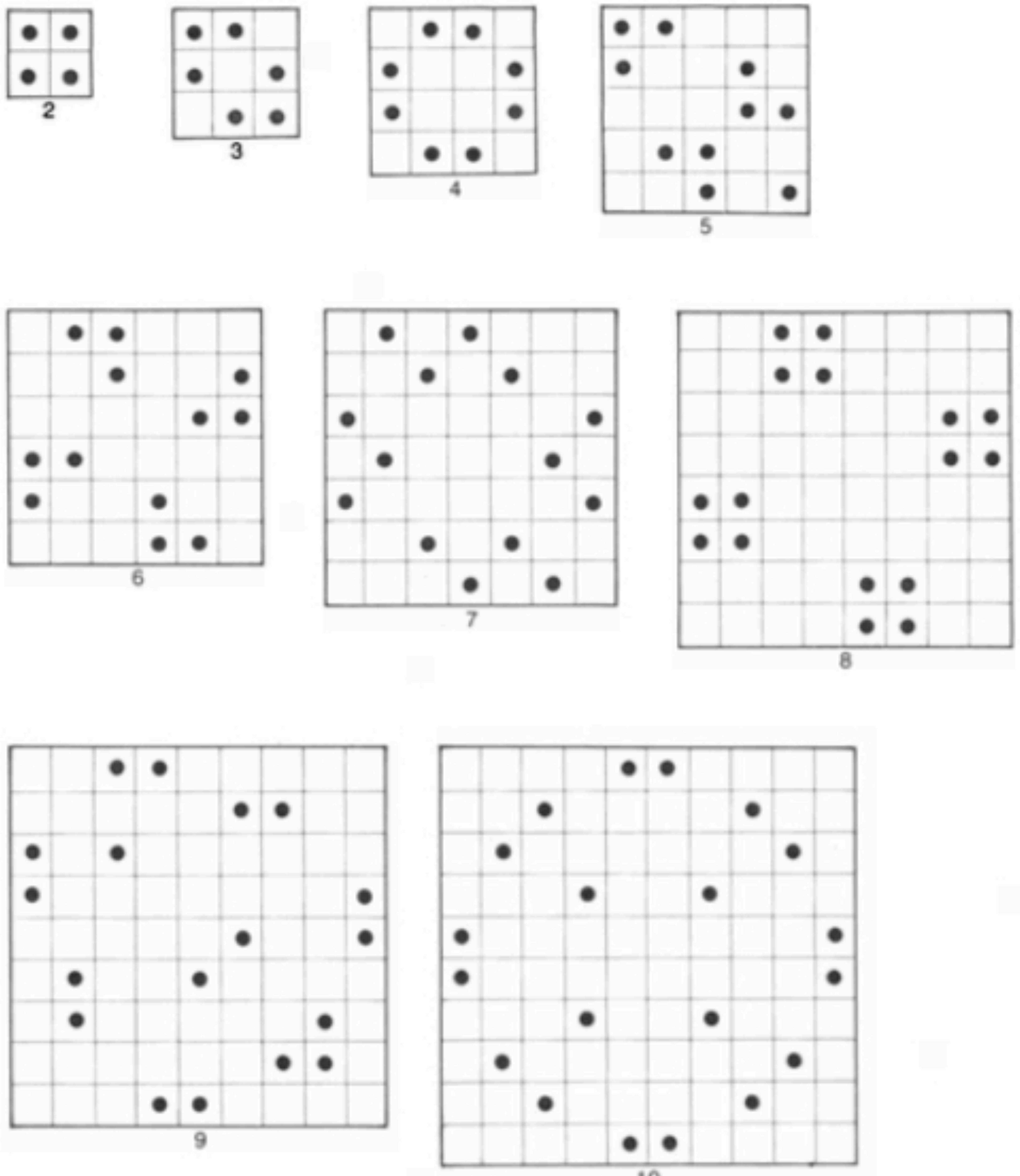


4 by 4 grid

place 8 pieces

no three in a line



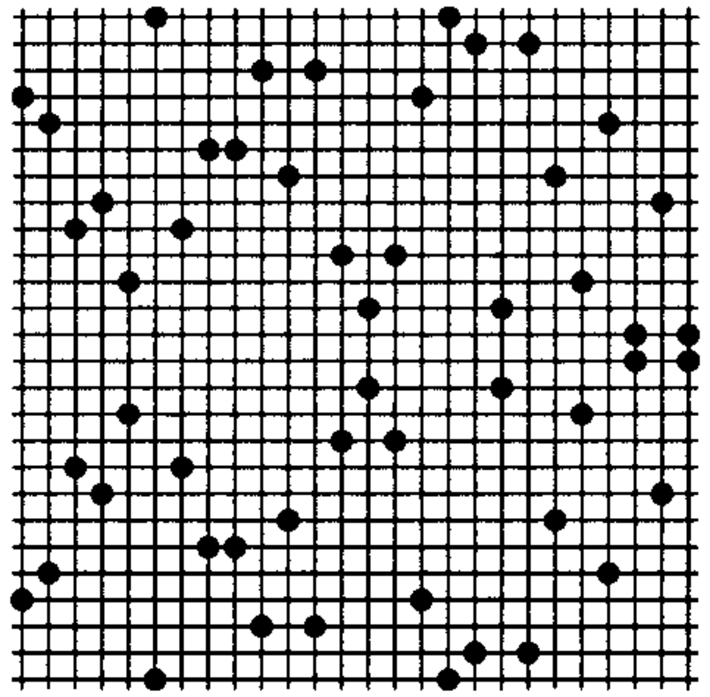


their paper "Some Advances in the No-Three-in-Line Problem" in *Journal of Combinatorial Theory*, Series A, Vol. 18, May, 1975, pages 336–341) proves that at least n counters can always be placed. For large boards these authors show that one can get quite close to $3n/2$ counters.

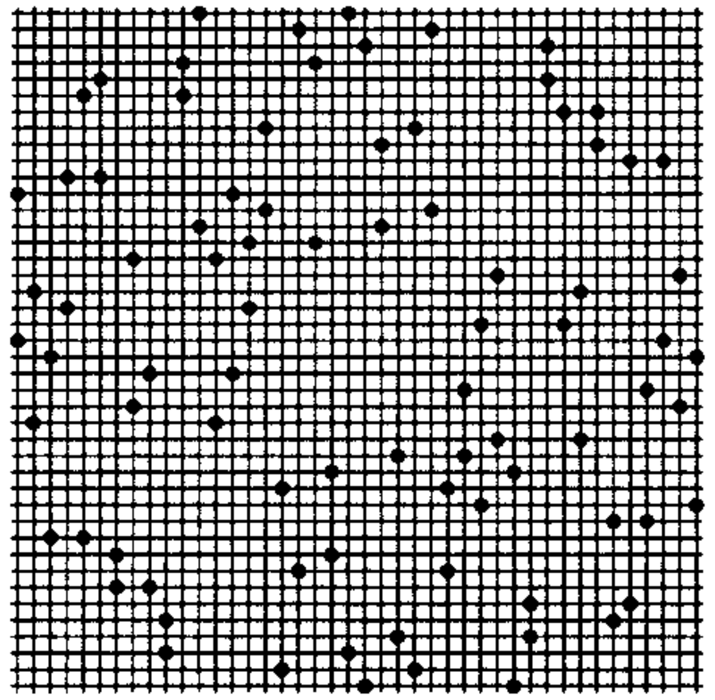
Michael A. Adena, Derek A. Holton and Patrick A. Kelly, in their paper "Some Thoughts on the No-Three-in-Line Problem" (in *Combinatorial Mathematics: Proceedings of the Second Australian Conference*, edited by Holton, Vol. 403 of *Lecture Notes in Mathematics*, Springer-Verlag, Berlin, 1974), reported on computer programs that found all distinct solutions for n equals 2 through 10. Rotations and reflections are excluded. The number of solutions are respectively 1, 1, 4, 5, 11, 22, 57, 51 and 156. The top illustration at the left gives an example for each n from 2 through 10. Note the startling simplicity and symmetry of the order-8 solution!

At the time these authors wrote, no solution for $n = 11$ was known. They gave a solution for $n = 12$, but it proved to be invalid. They had failed to notice two rows of three in line.

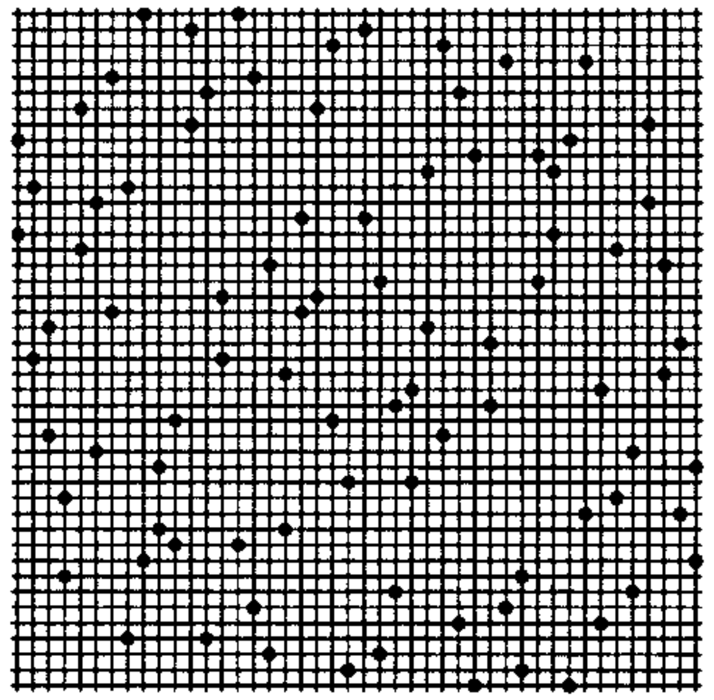
Solutions for $n = 11$ and $n = 12$ were found in 1975 by D. Craggs and R. Hughes-Jones of the University of Kent and published in *Journal of Combinatorial Theory* (Series A, Vol. 20, May, 1976, pages 363–364). They are shown in the bottom illustration at the left. Craggs and Hughes-Jones found five other solutions for $n = 11$ and three others for



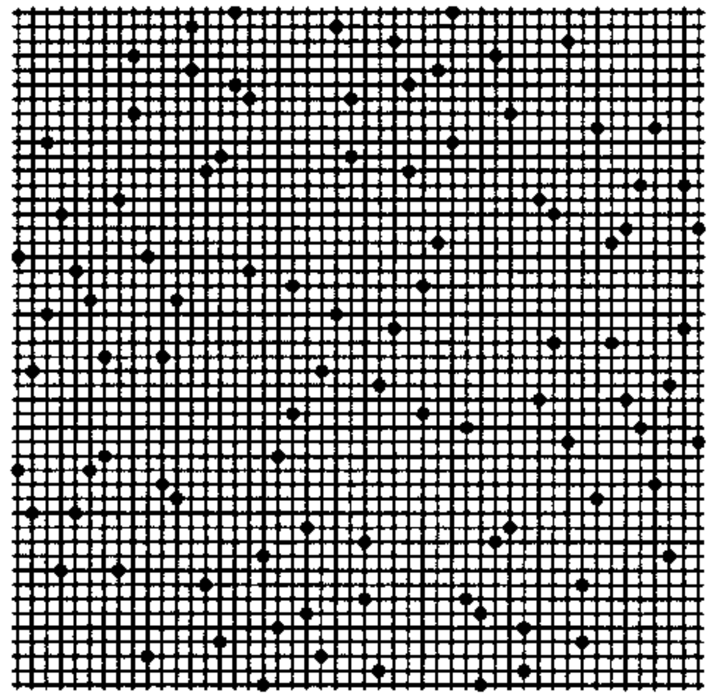
n=26 sym=ref1



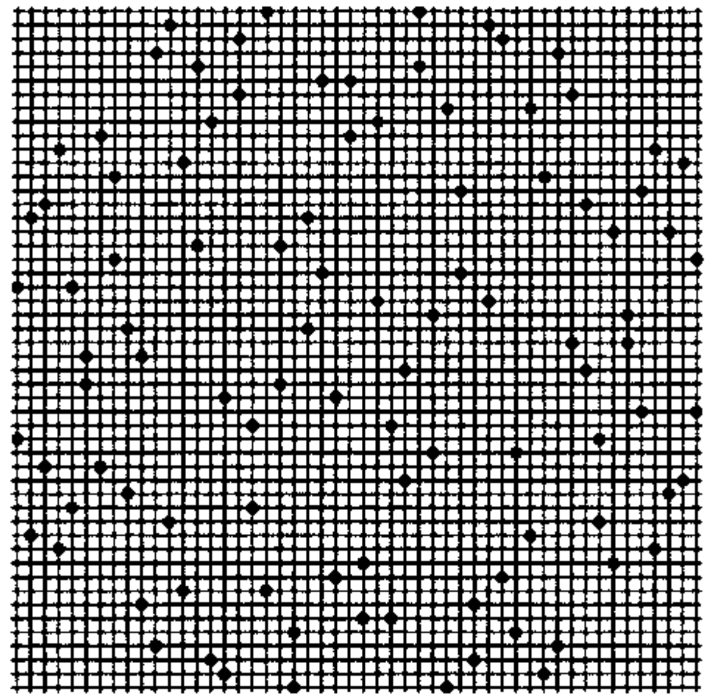
n=42 sym=dia2



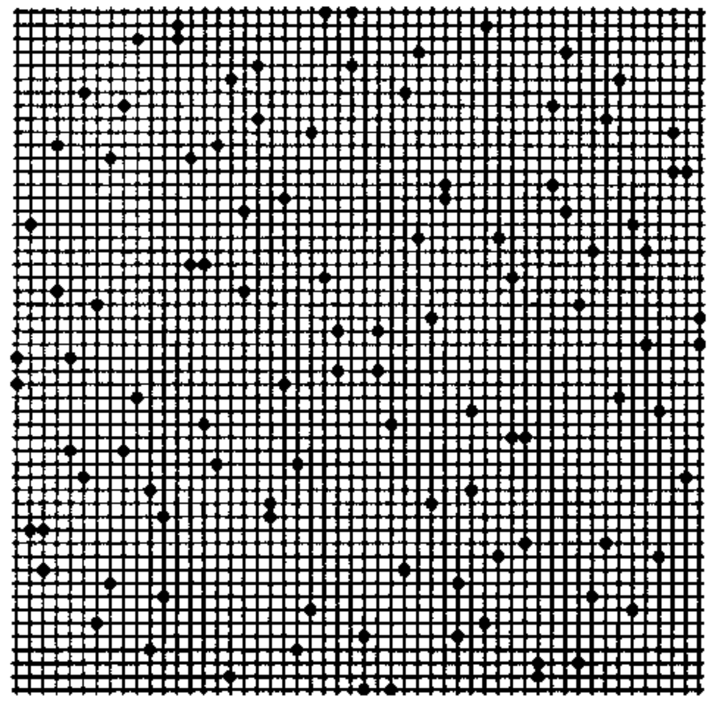
n=44 sym=dia2



n=48 sym=rot4

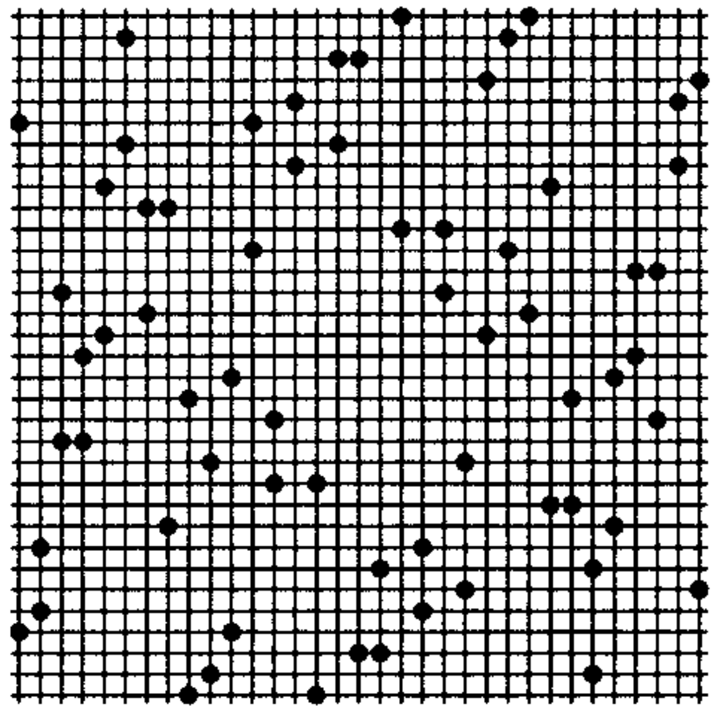


n=50 sym=rot4

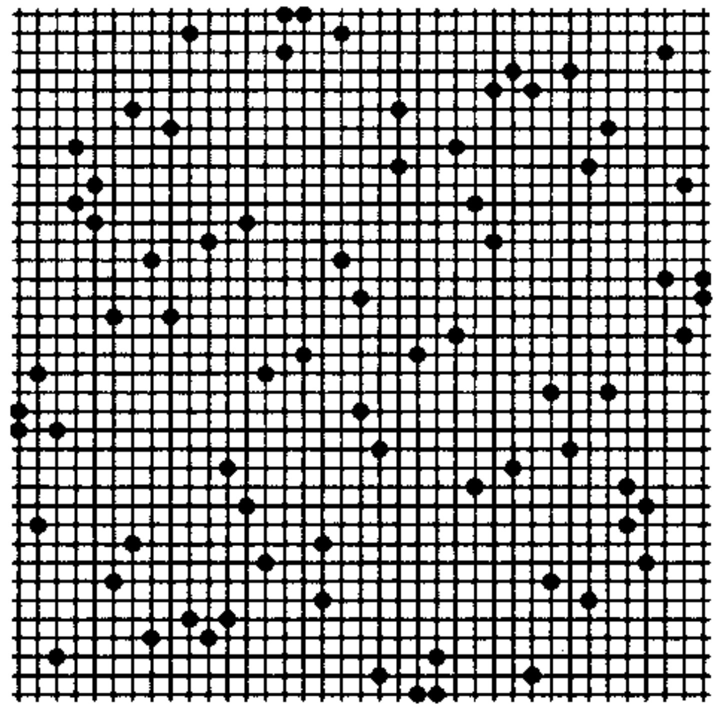


n=52 sym=rot4

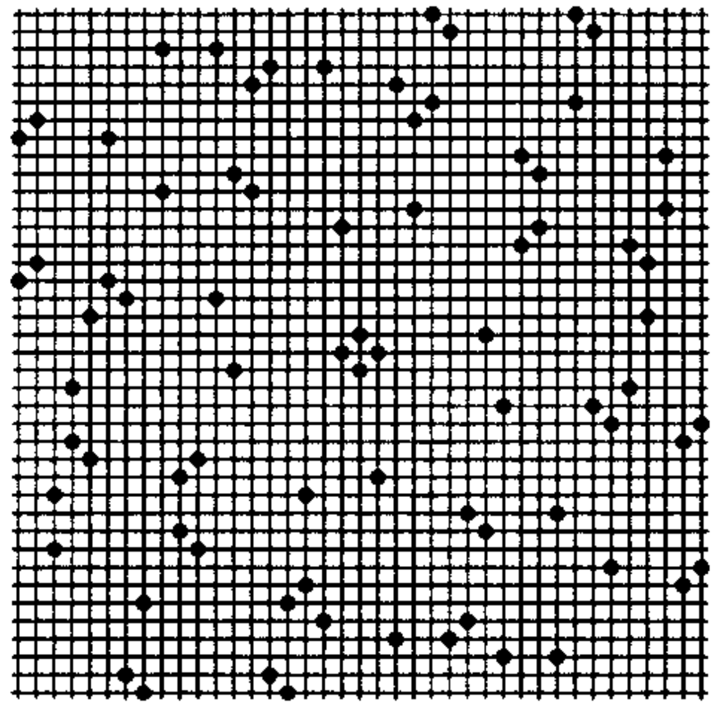
Fig. 1. Pictoral versions of some new configurations



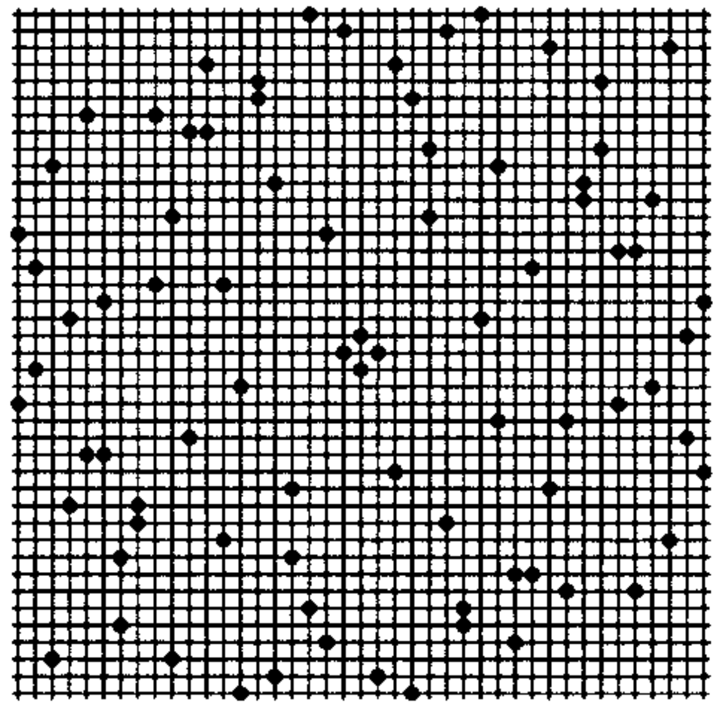
n=33 sym=rot2



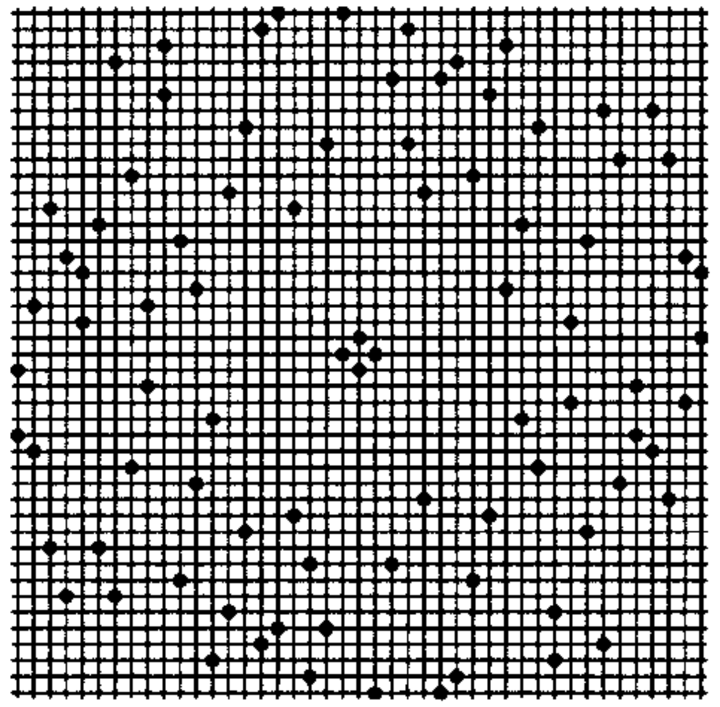
n=37 sym=near



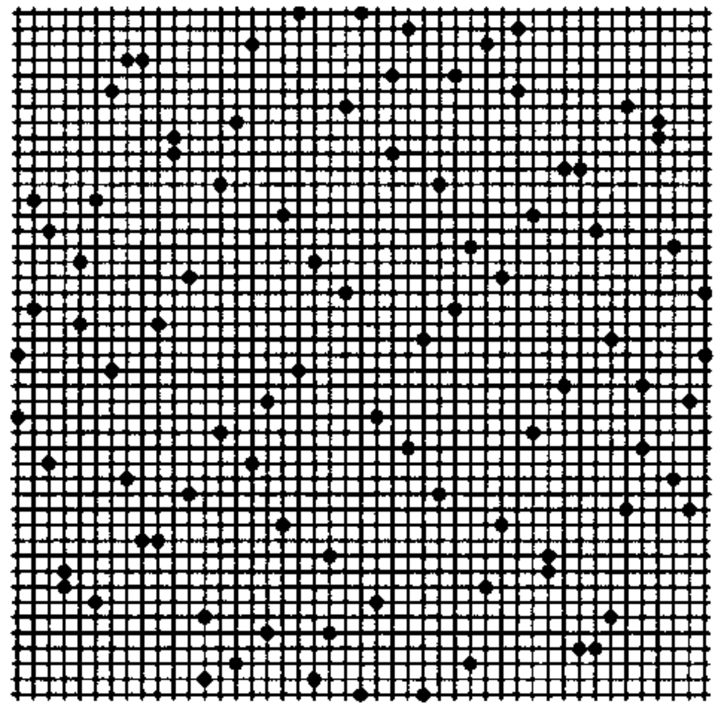
n=39 sym=near



n=41 sym=near



n=43 sym=near



n=45 sym=near

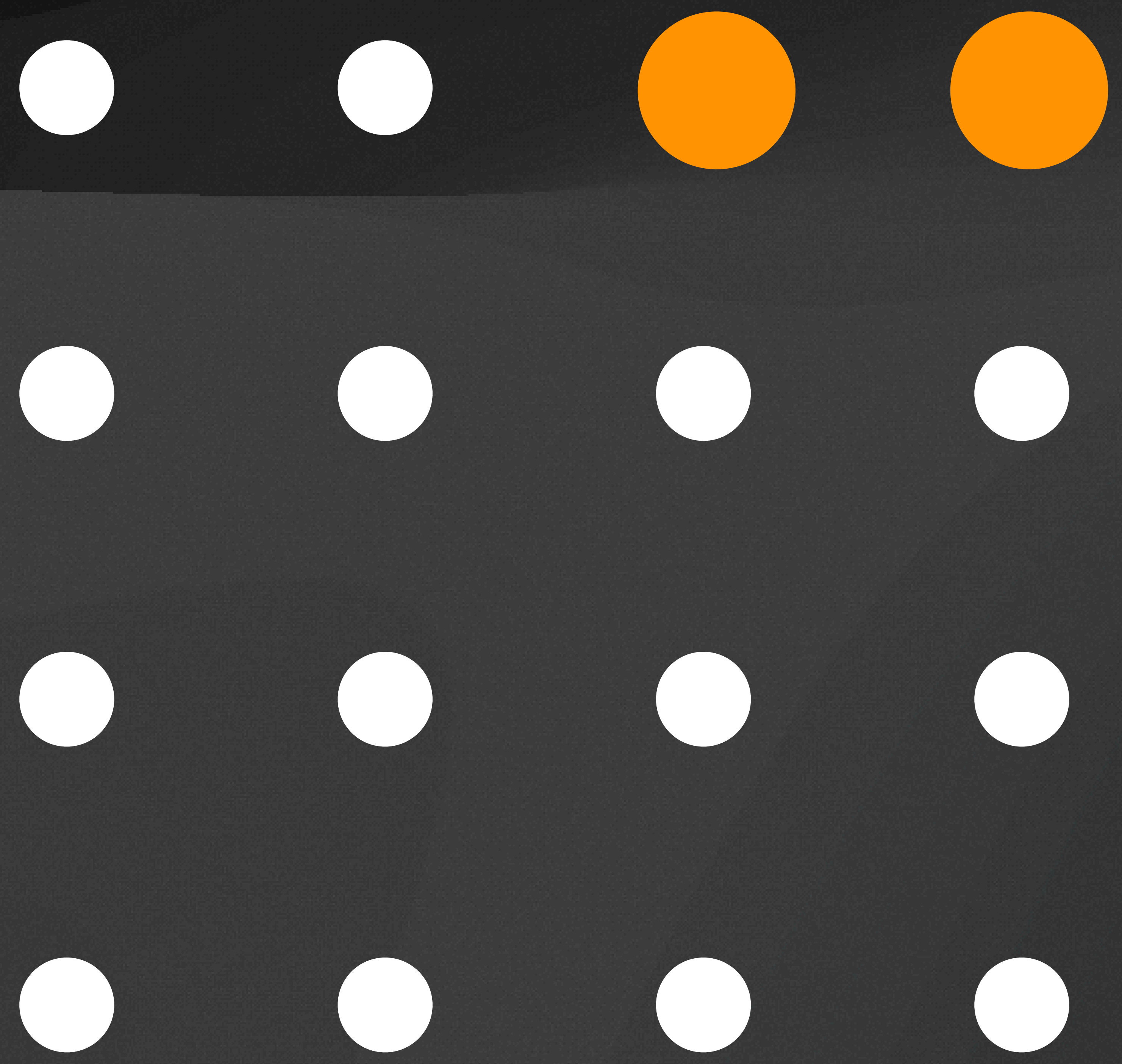
Fig. 1—*Continued.*

n	#ways	n	#ways
1	0	9	51
2	1	10	156
3	1	11	158
4	4	12	566
5	5	13	499
6	11	14	1366
7	22	15	3978
8	57	16	5900

oeis.org/A000769

can always place

$$\frac{3}{2}n$$



2n

$$\frac{\pi}{\sqrt{3}} \sim 1.8137\ldots$$

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