The Joy of Six!

Michael Gibson

6 is my favourite number

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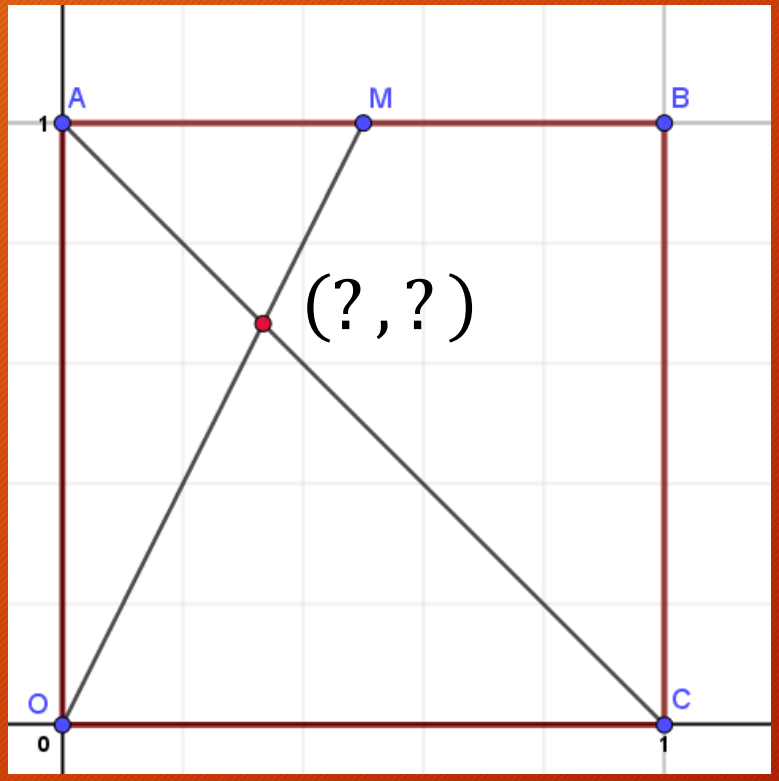
3!

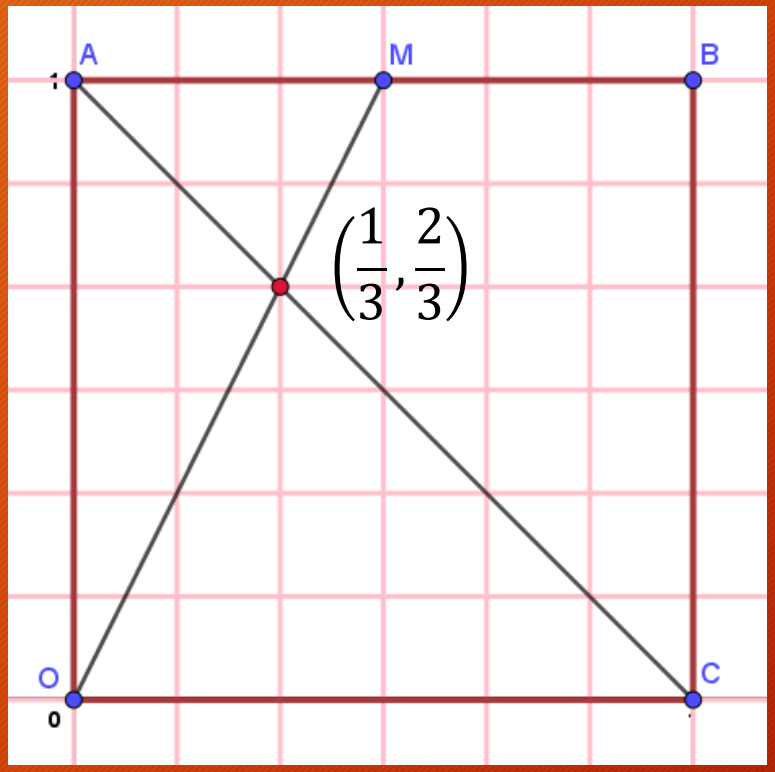
Warm Up!

- Square $OABC$ has vertices $O(0,0)$, $A(0,1)$, $B(1,1)$, and $C(1,0)$. Find the coordinates of the point of intersection between line segments AC and OM, where M is the midpoint of AB.

Warm Up!



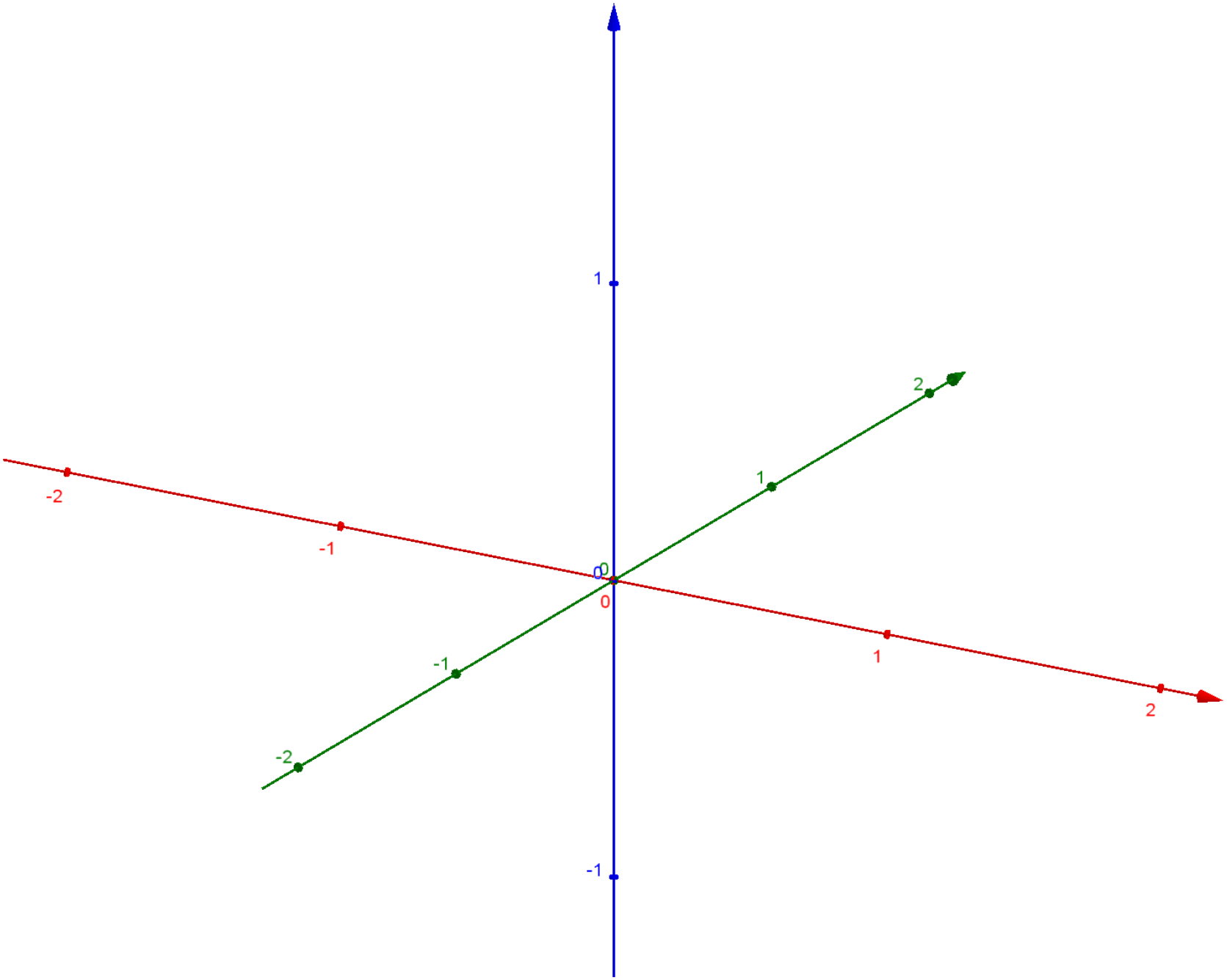
Warm Up!



STEP I 2007, Q5

i) Show that the angle between any two faces of an octahedron is $\arccos\left(-\frac{1}{3}\right)$.

ii) Find the ratio of the volume of an octahedron to the volume of the cube whose vertices are the centres of the faces of the octahedron.


1
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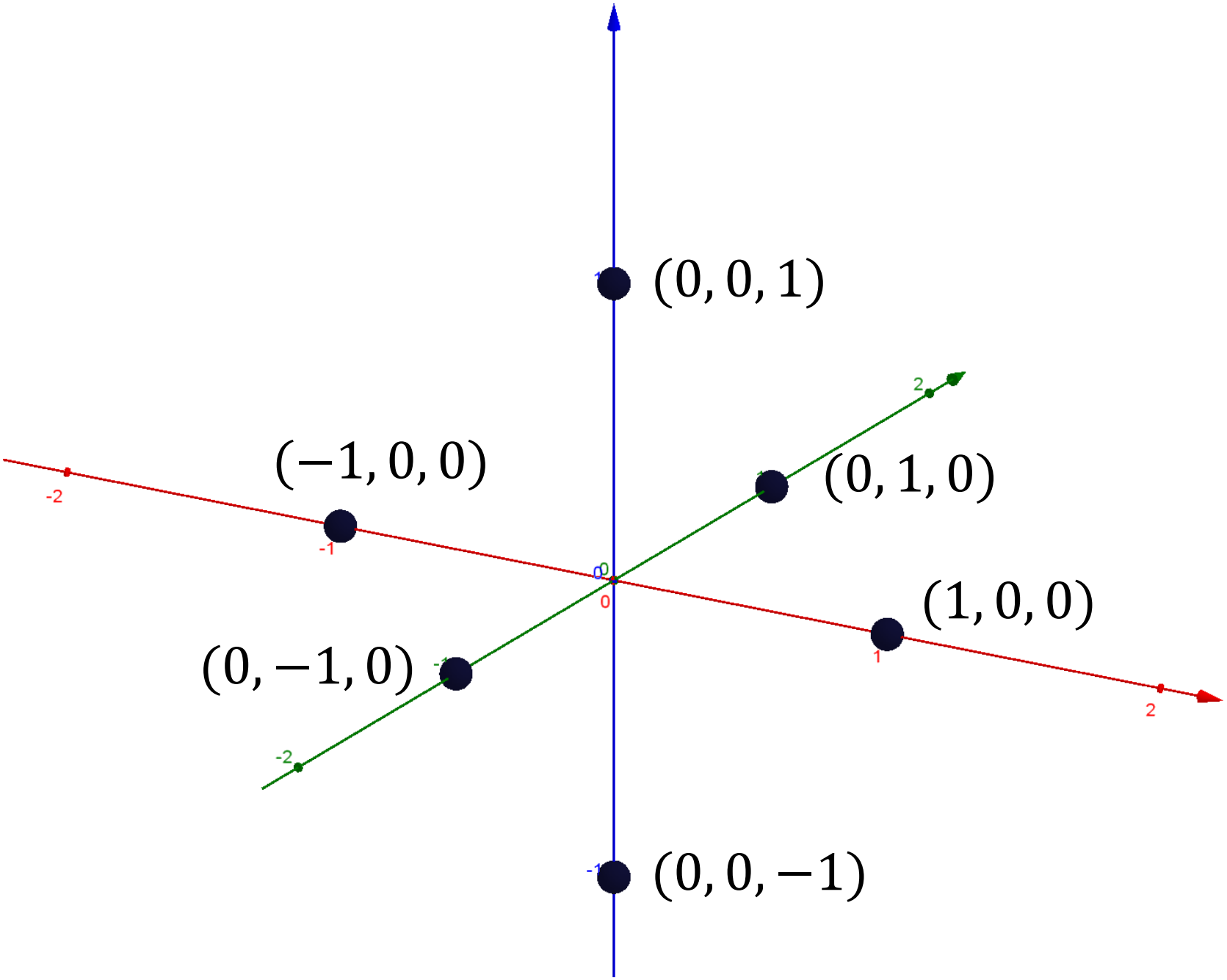
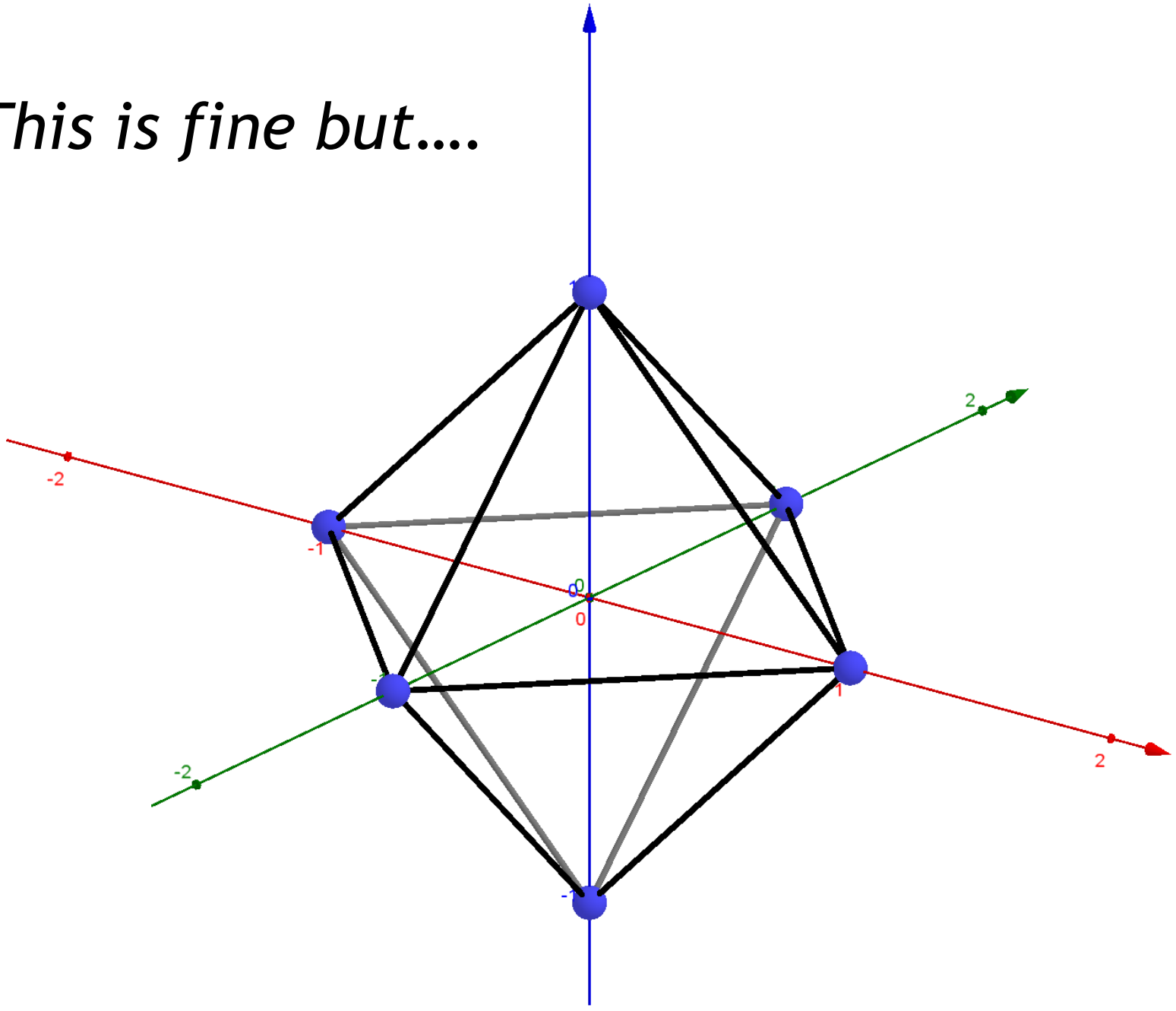
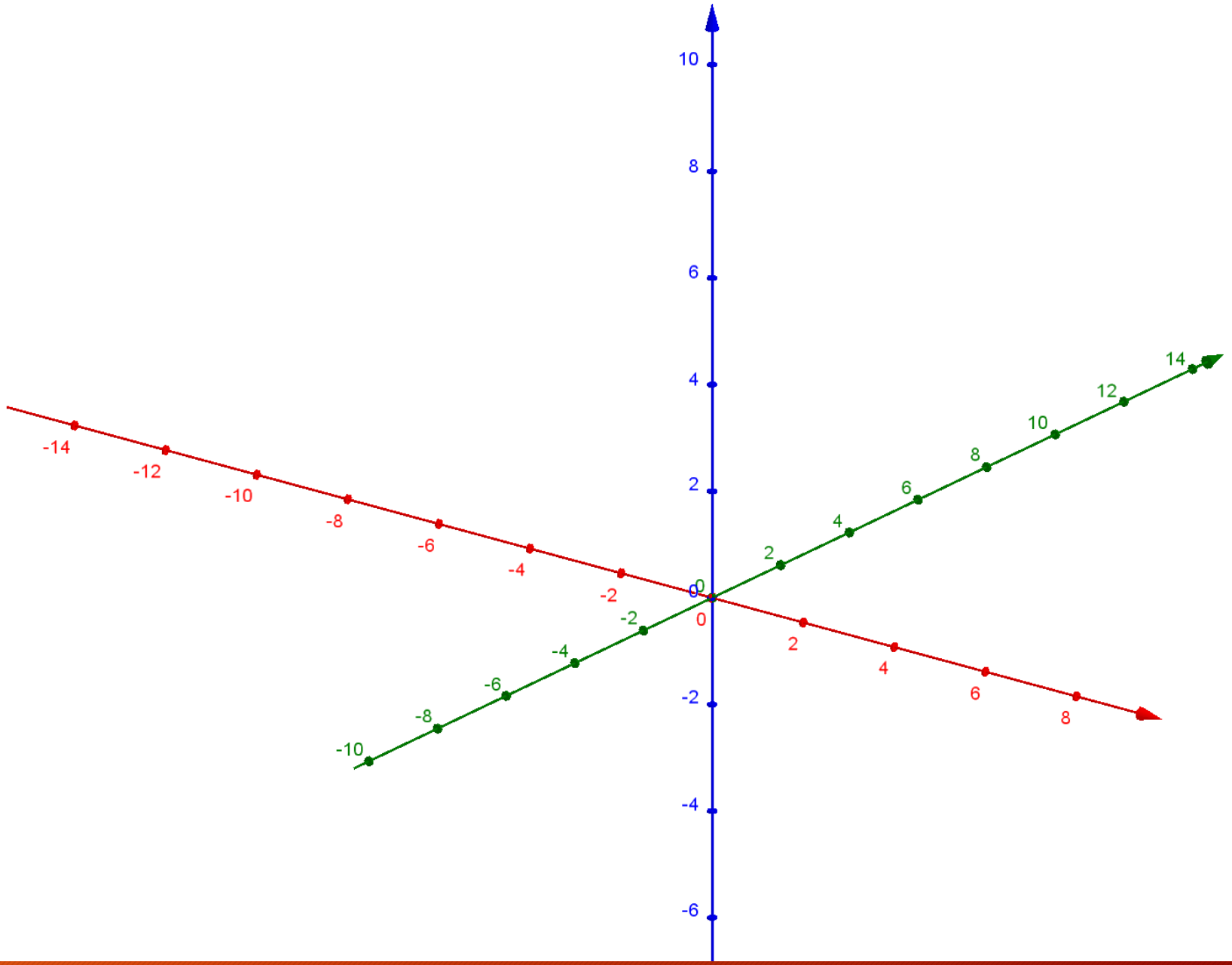

(0, 0, 1)
(−1, 0, 0)
(0, 1, 0)
(1, 0, 0)
(0, −1, 0)
(0, 0, −1)

This is fine but….

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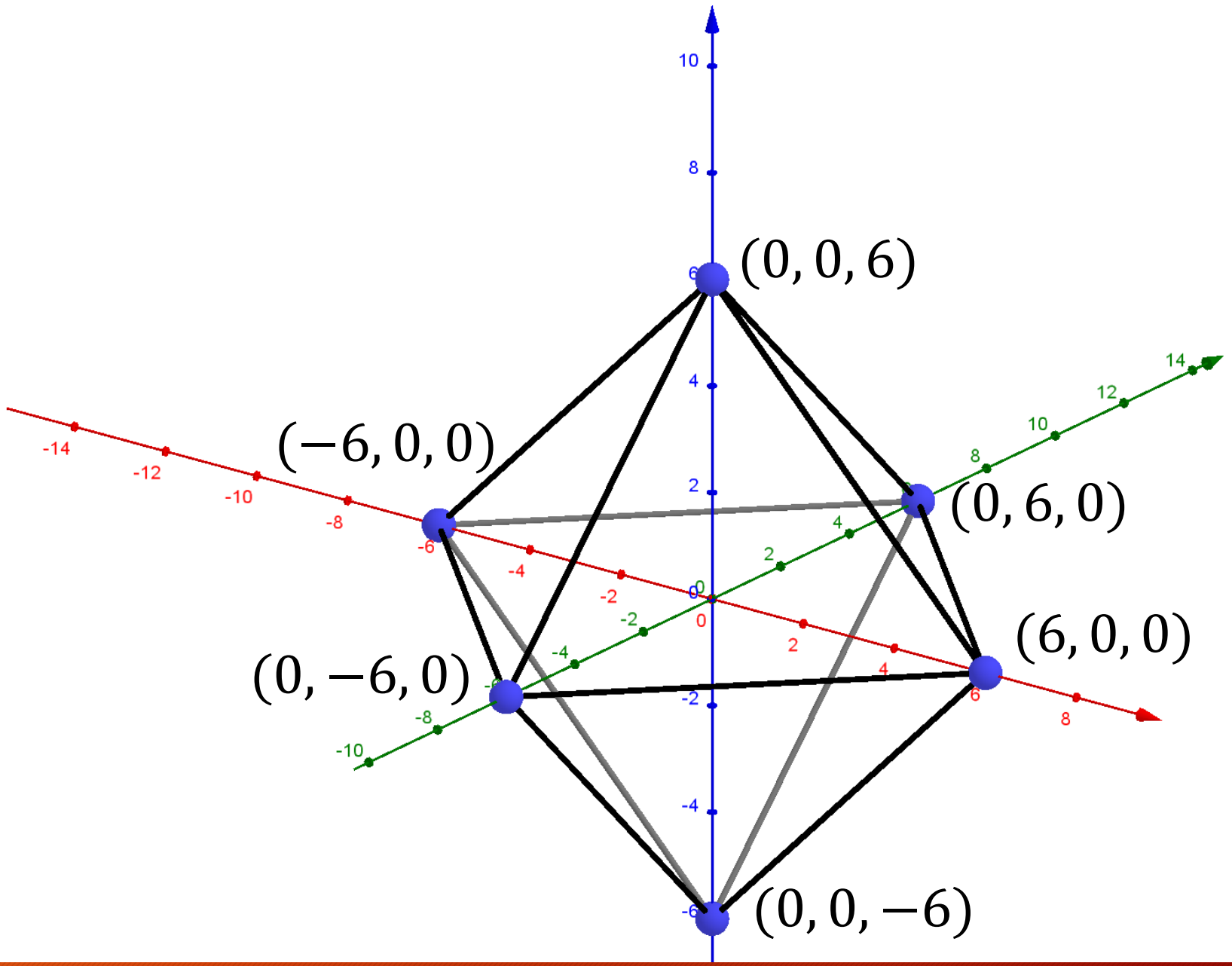
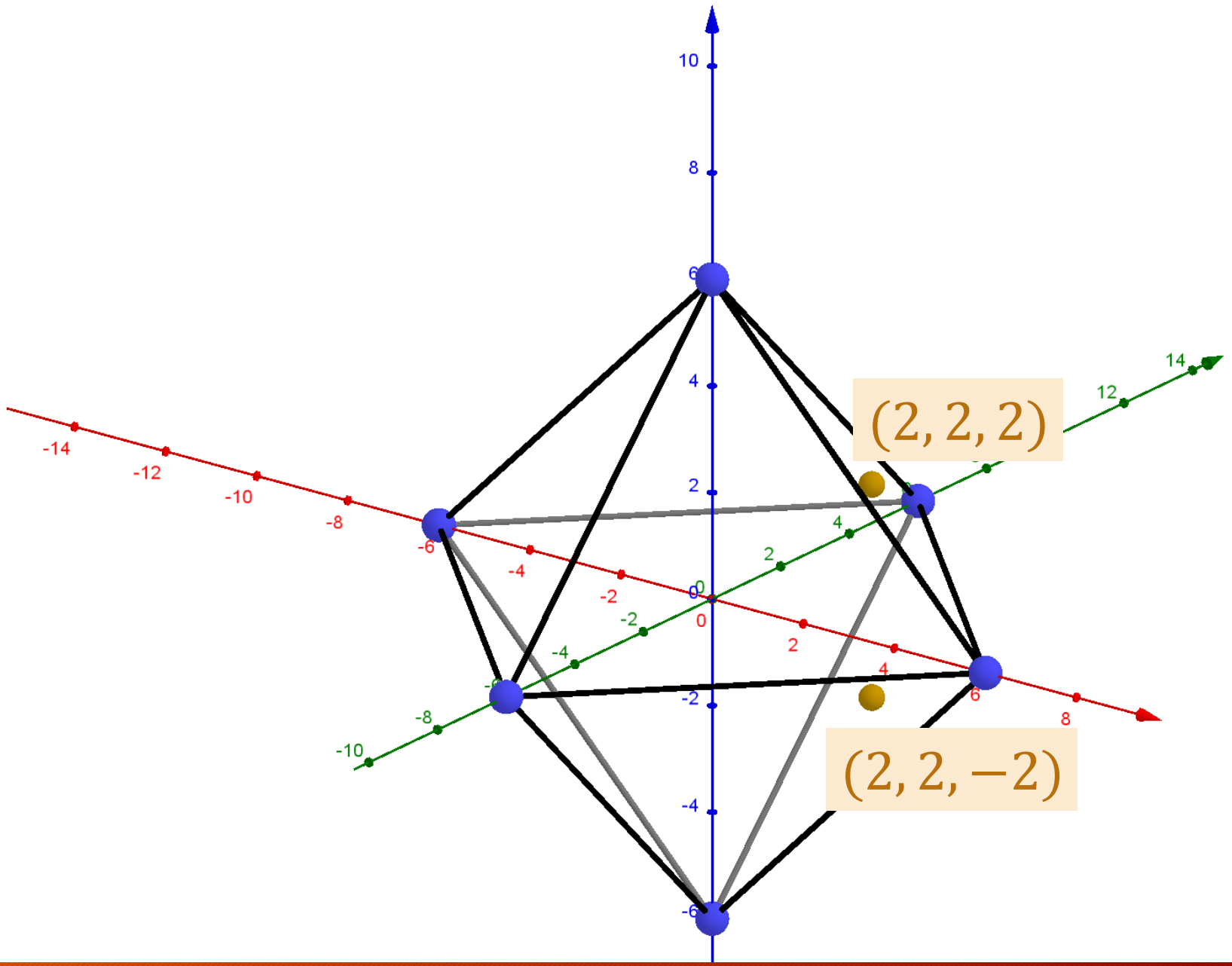
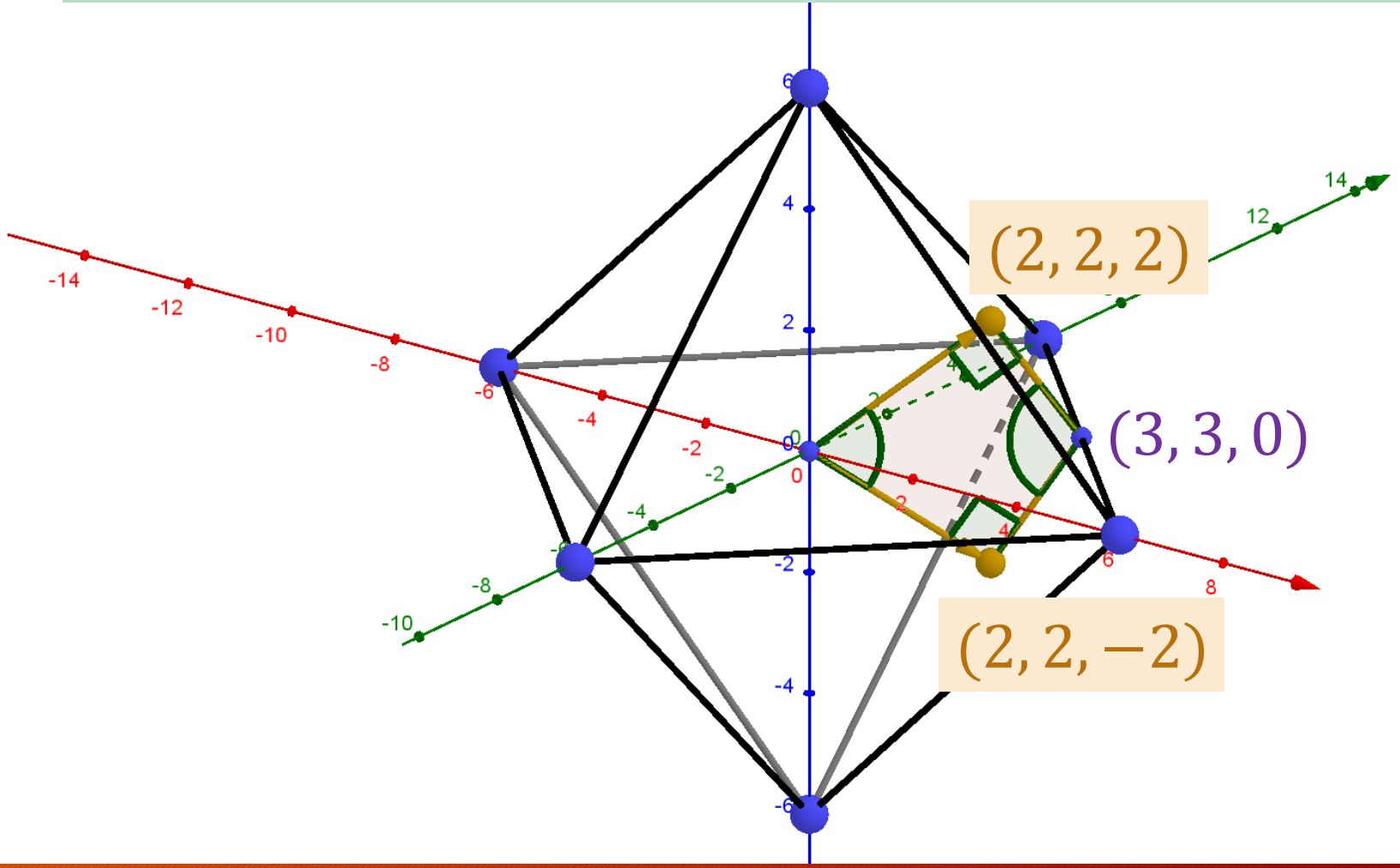
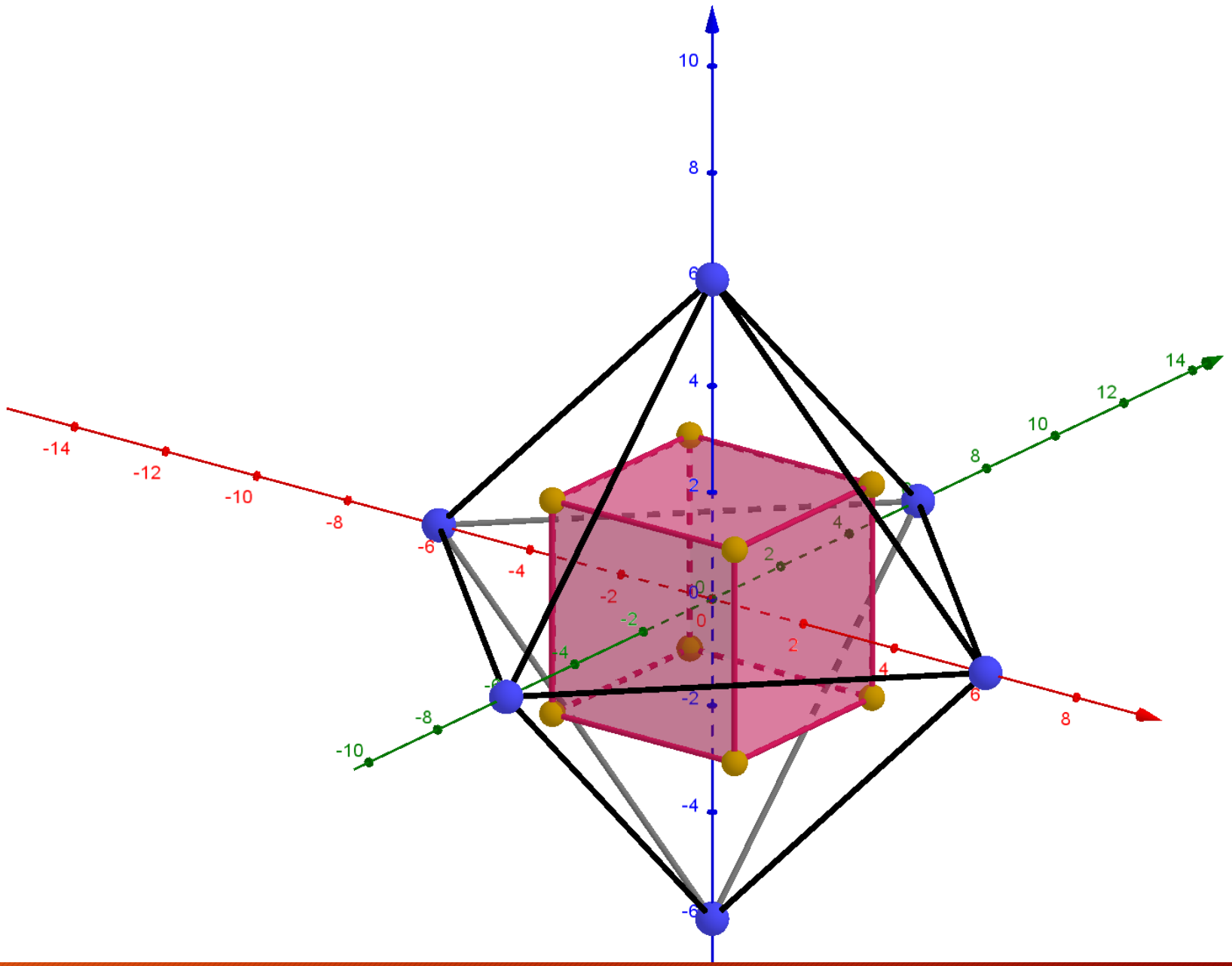

(0, 0, 6)
(−6, 0, 0)
(0, 6, 0)
(6, 0, 0)
(0, −6, 0)
(0, 0, −6)


(2, 2, 2)
(2, 2, −2)

$$\cos\theta = \frac{\begin{pmatrix}-1\\-1\\2\end{pmatrix}\cdot\begin{pmatrix}-1\\-1\\-2\end{pmatrix}}{\sqrt{1^2+1^2+2^2}\times\sqrt{1^2+1^2+2^2}} = \frac{-2}{\sqrt{6}\times\sqrt{6}} = -\frac{1}{3}$$

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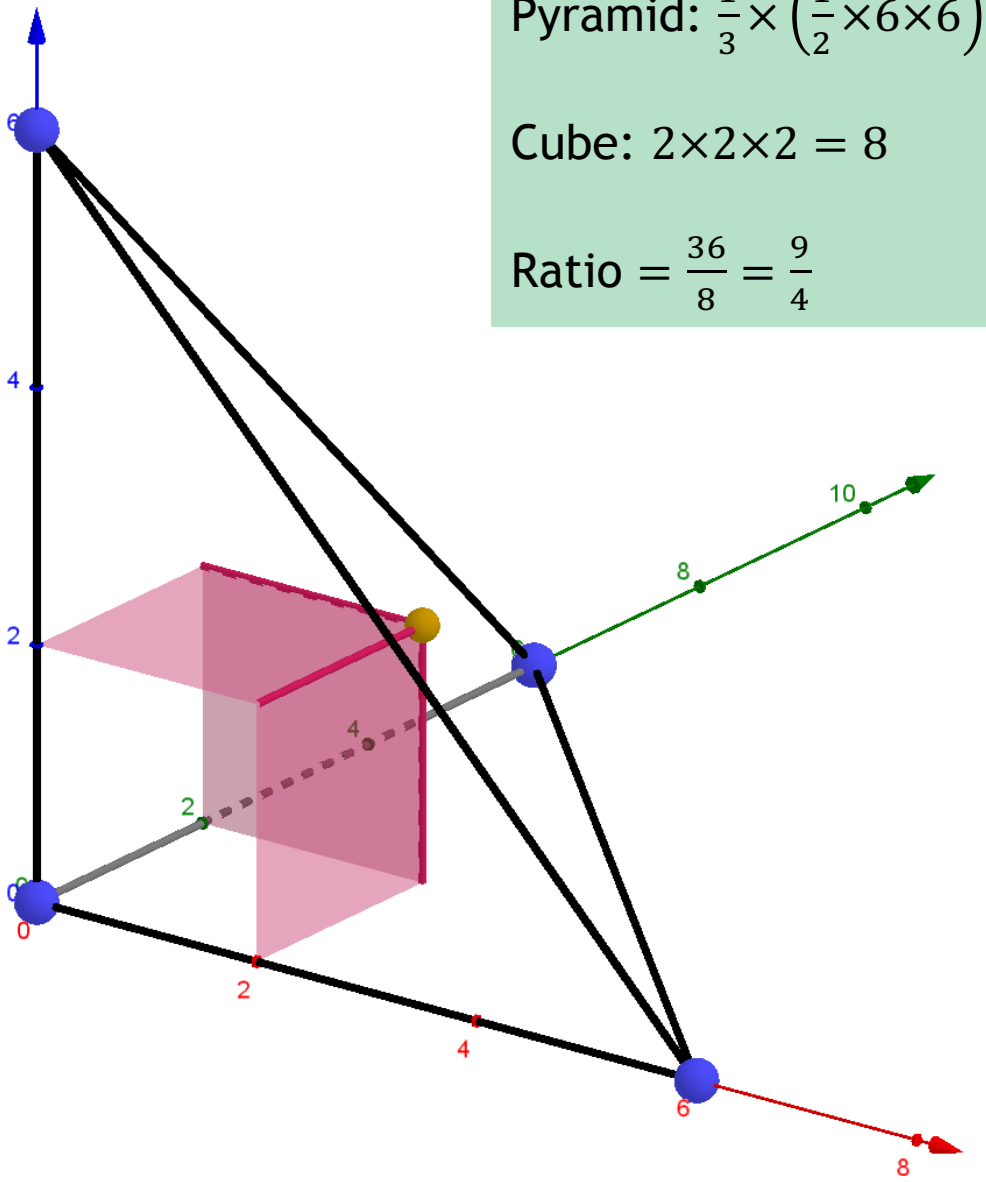
-8
-6
-4
-2
0
2
4
6
8

Pyramid: $\frac{1}{3}\times\left(\frac{1}{2}\times6\times6\right)\times6=36$

Cube: $2\times2\times2=8$

Ratio $=\frac{36}{8}=\frac{9}{4}$



6 is my favourite number

$$6 = 2 \times 3$$