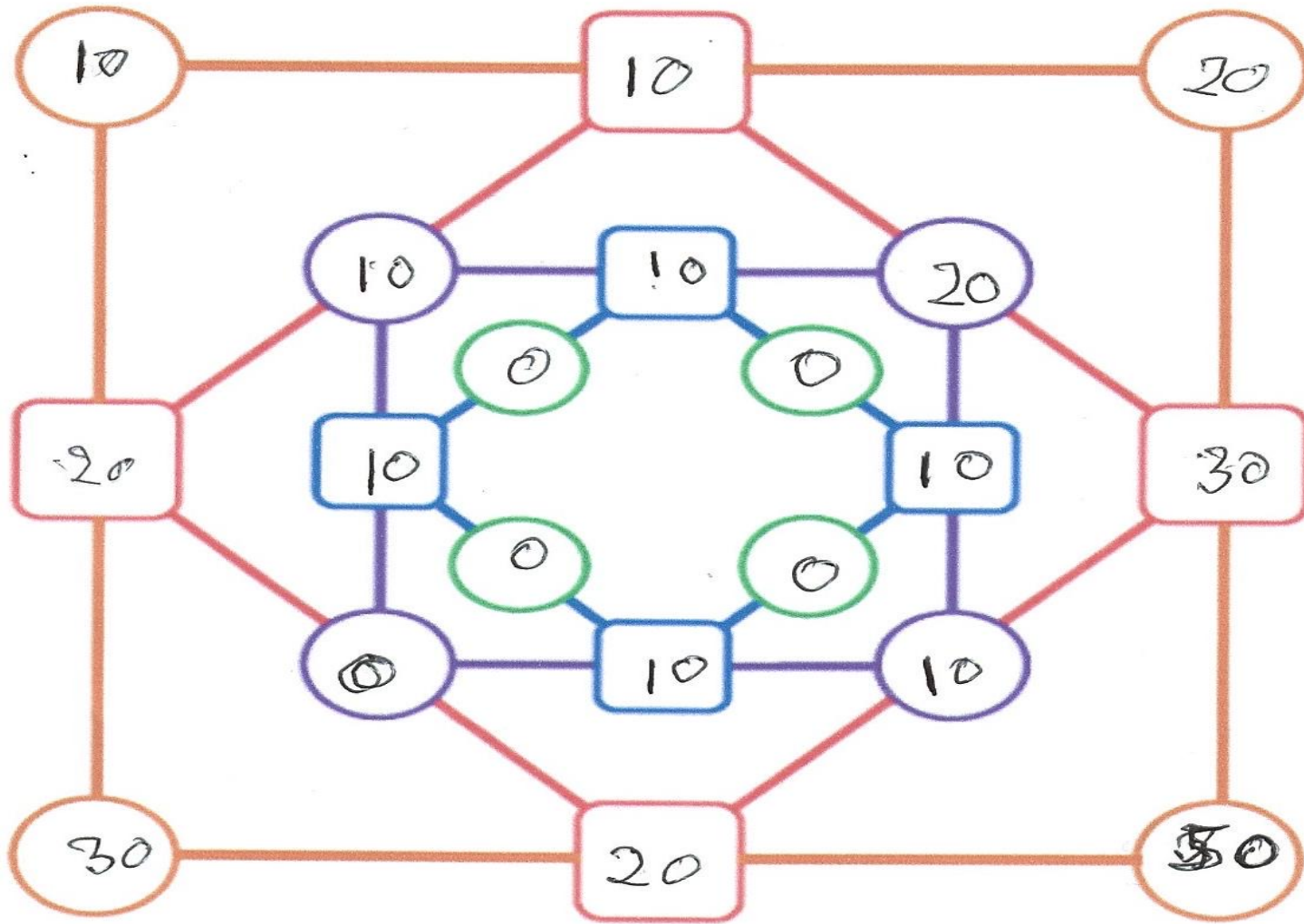


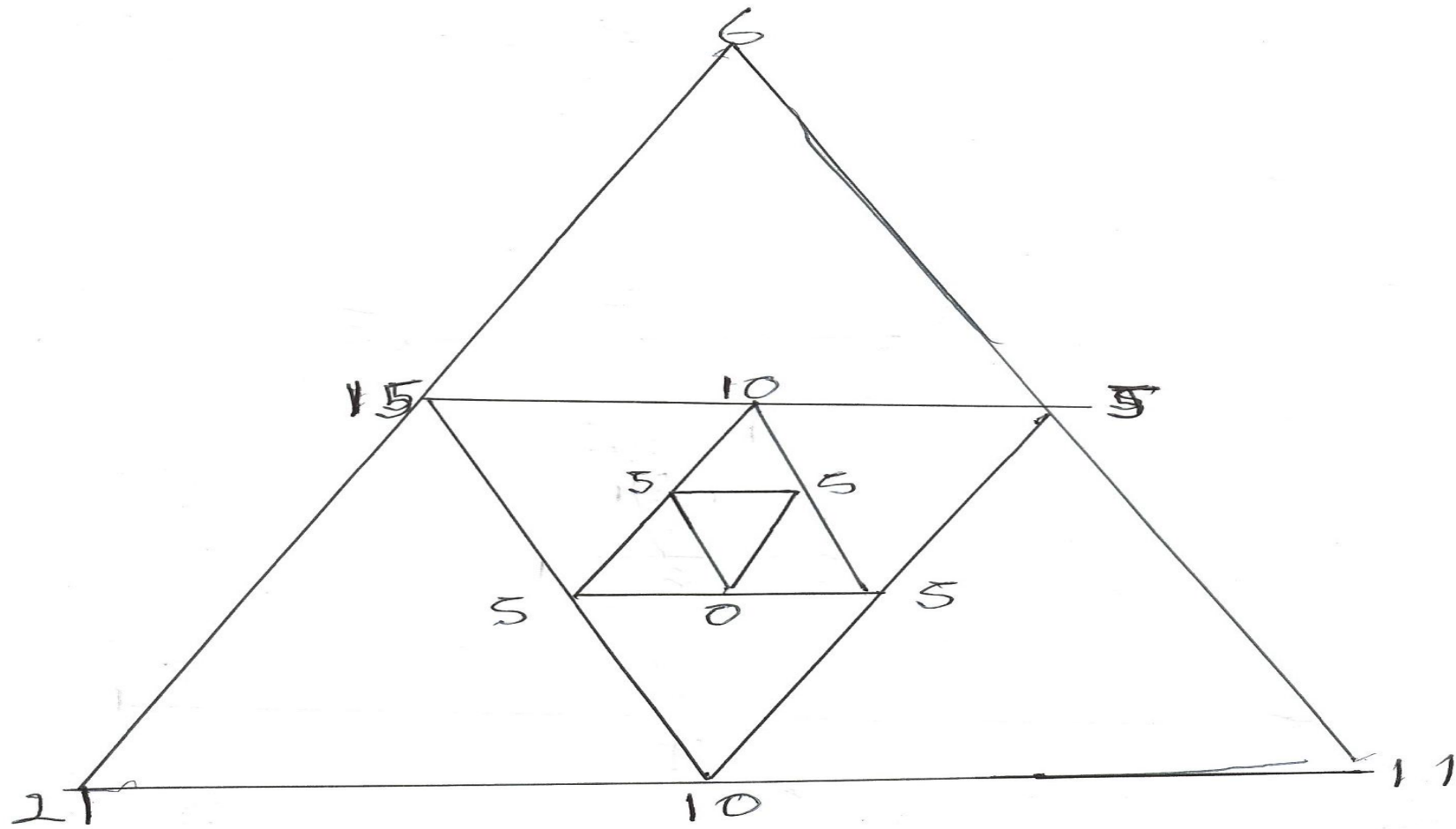
Diffy N-gons

What happens if we apply the rules and processes used for Diffy Squares to other polygons



Some Properties of a Diffy Square

1. All Diffy Squares reduce to the zero Diffy in a finite number of steps
2. The number of steps required depends on the starting values
3. For given starting values steps to the zero diffy is constant under addition and multiplication by a constant number and the symmetry of a square
4. A given a set of four distinct numbers can be up to 3 different number of steps depending on how they are arranged
(19,3,45,103) needs 5 steps, (3,19,45,103) needs 7 steps and (3,103,19,45) needs only 4 steps to reach the 0 diffy. Property 3 means that in this case there are 8 arrangements for each number of steps $3 \times 8 = 24$ the size of the permutation group of size 4



The Diffy Triangle

- Three equal numbers is the only case that reduces to the zero Triangle
This starting position I refer to as the Trivial Diffy.
- All other starting positions reach a cycle point
- The previous slide shows a starting position (6,11,21), it reaches (0,5.5) and then repeats for ever.
- Changing the starting numbers can change the number of steps to the cycle Point.

The Diffy 5-gon

- NB at this point I started using a spreadsheet so I could test lots of examples
- The trivial diffy goes to the zero diffy in one step
- All other values tested lead to a cycle step. The starting position (11,15,17,24,28) reaches a cycle point at step 12. (0,0,1,1,0) which repeats every 15th steps.
- So is this a feature of n-gons where n is odd What about a Diffy 7-Gon

The Diffy 7-Gon

- Apart from the trivial 7-gon all other starting positions tested result in a cycle point
- Example (13,27,45,101,201,302,351) reaches a cycle point in 42 steps (1,0,1,0,1,1,0) and repeats every 7 steps.
- So what about even n-gons
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The Diffy 6-gon

- There are some starting positions that result in the zero diffy the trivial diffy and what I have called the alternating diffy are two of them For example $(3,5,3,5,3,5)$ which in one step reduces to the trivial diffy $(1,1,1,1,1,1)$. Also $(0,10,20,10,0,10)$ reduces to the trivial diffy $(10,10,10,10,10,10)$. The common feature is that there is constant difference between each value. (The constant difference diffy)
- All others tested end in a repeating cycle. For example $(11,17,23,45,63,77)$ reach a cycle on step 19, $(0,0,0,4,0,4)$, repeats every 6 steps

The Diffy 8-gon

- All diffy 8-gons tried reduced to the 0-diffy
- Example
- Stating values, (4,11,31,71,82.83.101.201)
- Reduced to the 0-diffy in 19 steps
- The last two steps were the alternating diffy or a constant difference diffy followed by the trivial diffy
- So we always get the zero diffy for $n=4$ and $n=8$ what about other powers of 2.

The diffy 16-gon and 32-gon

Tried a number of different starting positions for each (thankful for the spreadsheet)

Every case reduced to the zero diffy

For all tried the last two steps were the alternating diffy or a constant difference diffy followed by the trivial diffy.

The Diffy 9-Gon

- Question given that diffy 4,8,16,32-gons reduced to the zero Diffy what about other prime powers.
- There is the trivial diffy, For odd n it is not possible to have a alternating diffy and no constant value diffy was found
- So all other diffy 9-gons tried always reached a cycle step
- For example (7,11,13,19,21,33,45,53,77) reaches a cycle in 26 steps (2,2,0,0,2,2,2,2,0) and then repeats every 63 steps.

So the main conjectures so far is:-

- For all diffy n-gons where n is a power of 2 all starting positions will result in the 0-diffy
- For odd n the only the trivial diffy will reduce to the 0-diffy
- For even n not a power of 2 only the alternating diffy the constant value diffy and the trivial diffy will result in the 0-diffy. All others reach a cycle point

Some Next steps

- Investigate the relationship of the symmetry of the n-gons to the permutation groups and how the starting values and the arrangement of the starting values affect the steps to the 0-diffy or the cycle point.
- The constant diffy for the 6-gon (0.10.20.10.0.10) used 3 different values. Can constant distant diffys be found using other values
- Prove the result for powers of 2