

A 19th century lottery – with a deck of cards

Pedro Freitas and Jorge Nuno Silva
FCUL and CIUHCT

MathsJam Gathering
November 11 and 12, 2023



CIUHCT

Centro Interuniversitário de História
das Ciências e da Tecnologia
FCUL | FCT - UNL



Ciências
ULisboa

História e Filosofia
das Ciências

FCT

Fundação
para a Ciência
e a Tecnologia

Supported by FCT, I.P. through projects UIDB/00286/2020 and UIDP/00286/2020

Francisco Giraldes Barba

Francisco Barba (June 5, 1780 - April 15, 1855) was a Portuguese scholar and nobleman, with a military career and a corresponding member of the Portuguese Academy of Sciences.

In 1834, he published an essay, with a lottery plan. The main novelty of the plan was that it was implemented with a deck of cards and could be played as a social recreation.

Probably inspired by Ozanam's *Récréations mathématiques* (1770).



The lottery

The main ideas:

- Two decks are used, one for the Bank and another one to be distributed among the Players.
- *A lottery ticket* is a triplet of cards.
- Each Player must pay an entry for each ticket they hold.
- According to the cards drawn by the Bank, some cards will be null, some will have a small prize (equal to the value of the entry), and some will have a big prize.
- The prizes are designed to assure a 12% profit to the Bank.

The lottery plan

The *simple version*:

- The Players divide a full deck among themselves. Each one pays one unit for each triplet: if a player has n cards, they must pay n choose 3 units.
- The Bank shuffles their deck and draws the first 6 cards: these are declared null, and any triplet that includes any of these cards has no prize. All others have, at least a *small prize*, equal to one unit.
- The last 10 cards of the Bank's deck will determine the *large prizes*. Any triplet formed with these cards has a large prize, the value of the prize increasing as the cards approach the end of the deck.

For instance, a triplet formed with the 44th, 46th and 48th cards will earn a prize of 20 units, whereas a triplet formed with the last three cards (50th, 51st and 52nd) earns a prize of 538.

An example

- Your cards: A♣ 3♣ 10♦ Q♦ K♦ A♠ 2♠ A♥ 2♥

You must pay $\binom{9}{3} = 84$ units (this is how many tickets = triplets you have).

- Null cards: A♣ 4♣ 5♣ 6♦ K♠ Q♥

One of your cards is null, so you only get back $\binom{8}{3} = 56$ small prizes (equal to units, you lost 28).

- Prize cards (from 43 to 52, in order): 10♠ 3♥ 10♦ 9♠ 9♦ 3♦ 4♦ K♦ 3♥ 3♣

You have one large prize. Since the first card drawn in this triplet was the 45th, you get a large prize of 30 units (this value is taken from a table).

In the end, you win a total prize of 2 units.

Documents

In this implementation, not all triplets are in play in a game, and not all prizes are given.

However, the author suggests a way of accomplishing this.

The author establishes an order in the deck and defines the *document of a card* as the set of all triplets formed by that card and two preceding ones.

The main feature:

Documents are a partition of the set of triplets

In document play, each card held by a Player represents its document.

He suggests that only a half deck can be used, so that the numbers do not get too big.

The author has many tables to help with computations.

An exemple with 26 cards

- Your cards: A♣ 3♣ 5♣ 4♦ 6♦ (in order)

- Null cards: 4♣ 3♦ 4♦ Q♦

The 4♦ generates blank tickets in the documents 4♦ (all) and 6♦ (some). Same for 4♣ and 3♦.
Total: 180 blank tickets.

- Prize cards (from 17 to 26, in the order they were drawn): 3♣ K♣ J♦ A♠ 2♣ 5♦ 6♣ 6♦ 5♣ A♣

Document 6♦ has 21 prizes, document 5♣ has 3 prizes and document 3♣ has one prize.
Total: 328 units.

In the end, you win a total prize of $328 - 180 = 148$ units

NOVO E CURIOSO JOGO
DE
CARTAS OU NUMEROS,
DENOMINADO
LOTÉRIAS LISBONENSES.



LISBOA:
NA IMPRENSA NACIONAL.
1834.
Com Licença.

As Loterias Lisbonenses
(1834) de Francisco António
Marques Giraldes Barba



Jorge Nuno Silva