9999713179999 is prime

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MathsJam

2 November 2024

- Katie Steckles spoke about emirps on a Finite Group livestream.

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e.g.

$$13$$
$$3541$$
$$9999713$$

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e.g.

$$13 \to 31$$
$$3541 \to 1453$$
$$9999713 \to 3179999$$

Does it always work?

19

Does it always work?

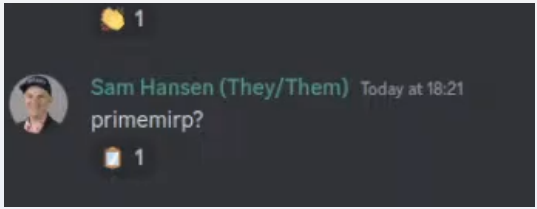
$$19 \longrightarrow 91$$

Does it always work?

$$19 \rightarrow 91 = 7 \times 13$$

- No!

- In the comments...



Overlapping the middle digit. Why?

- For prime $a_1 a_2 \ldots a_n$ and emirp $a_n \ldots a_2 a_1$.

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- Since

$$10^k \pmod{11} = \begin{cases} 10, & \text{if } k \text{ is even;} \\ 1, & \text{otherwise,} \end{cases}$$

each a_i contributes a multiple of eleven (i.e. not prime).

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  each a_i contributes a multiple of eleven (i.e. not prime).
- An overlapping middle digit ('prime<u>e</u>mirp' rather than 'prime<u>ee</u>mirp') avoids this by making the pair of a_i either odd or even but not both.

primemirp

- Does it work?

$$13$$

$$907$$

$$9103$$

primemirp

- Does it work?

$$\underline{13} \quad \& \quad 31$$
$$\underline{907} \quad \& \quad 709$$
$$\underline{9103} \quad \& \quad 3019$$

primemirp

- Does it work?

$$\underline{13} \;\&\; 31 \;\rightarrow\; \underline{131}$$
$$\underline{907} \;\&\; 709 \;\rightarrow\; \underline{907}09$$
$$\underline{9103} \;\&\; 3019 \;\rightarrow\; \underline{9103}019$$

primemirp

- Does it work?

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$$\underline{907} \;\&\; 709 \;\rightarrow\; \underline{907}09$$
$$\underline{9103} \;\&\; 3019 \;\rightarrow\; \underline{9103}019$$

- Yes!

primemirp

- Does it always work?

$$17$$

primemirp

- Does it always work?

$$17 \;\&\; 71$$

primemirp

- Does it always work?

$$\underline{17} \ \& \ 71 \rightarrow \underline{171}$$

primemirp

- Does it always work?

$$\underline{17} \ \& \ 71 \rightarrow \underline{171} = 9 \times 19$$

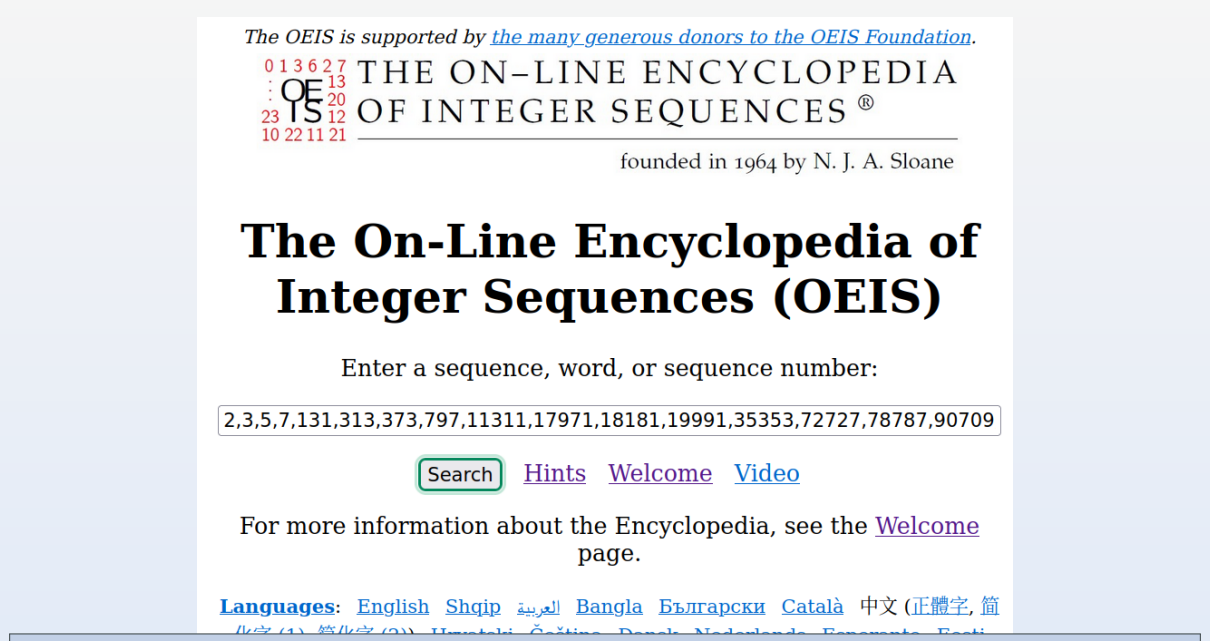
- No!

CAN HUMANS SAY THE LARGEST PRIME NUMBER BEFORE WE FIND THE NEXT ONE?

saytheprime.com

- I wrote some code which:
  - found primes;
  - reversed them, checked whether the reversal was prime;
  - concatenated the primes and emirps and checked if they were prime.
- I found a few!

```
for i in range(2,10**7):
  if isprime(i):
    emirp = int(str(i)[ 1:: 1])
    if isprime(emirp):
      primemirp = int(str(i)+str(emirp)[1:])
      if isprime(primemirp):
        print(primemirp)
```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2,3,5,7,131,313,373,797,11311,17971,18181,19991,35353,72727,787 Search

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

Search:
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Displaying 1-1 of 1 result found. page 1

Sort: relevance | references | number | modified | created Format: long | short | data

A054218 Palindromic primes of the form 'primemirp' resulting from A054217. +30 3

2, 3, 5, 7, 131, 313, 373, 797, 11311, 17971, 18181, 19991, 35353, 72727, 78787, 90709, 93739, 96769, 98389, 1153511, 1193911, 1201021, 1409041, 1583851, 1597951,

primemirp

- Added to OEIS in February 2000 by Patrick De Geest.
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primemirp

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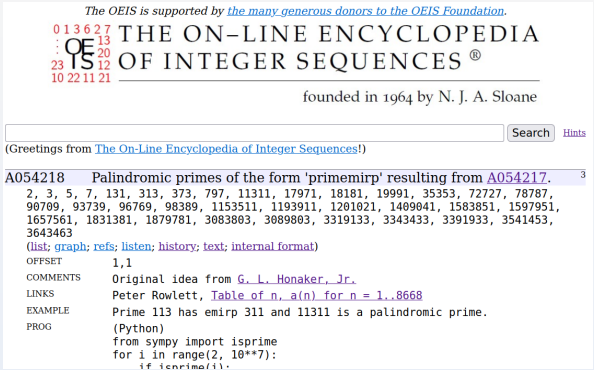
- Based on an idea from G. L. Honaker, Jr.
- But there was no code, and only 32 examples were given.
- I edited the entry to add a Python program and entries up to the 8,668th, which I believe is all primemirps where the underlying prime is less than ten million.
- The 8,668th primemirp is 9,999,713,179,999.



Thanks for listening!