

9999713179999 is prime

Peter Rowlett

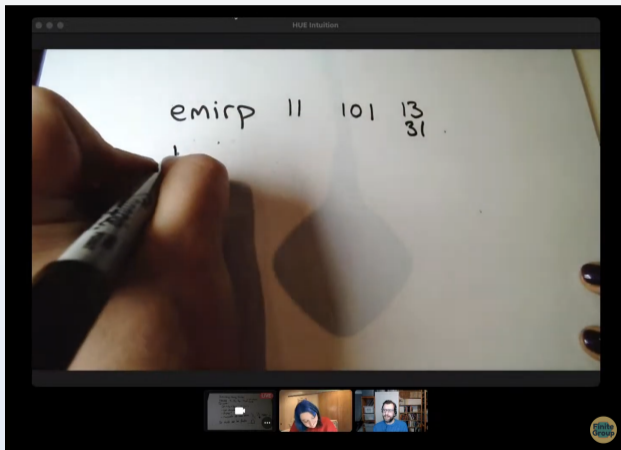
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MathsJam

2 November 2024

- ▶ Katie Steckles spoke about emirps on a Finite Group livestream.



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e.g.

13

3541

9999713



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- ▶ Katie Steckles spoke about emirps on a Finite Group livestream.

e.g.

$$13 \rightarrow 31$$

$$3541 \rightarrow 1453$$

$$9999713 \rightarrow 3179999$$



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Does it always work?

19

Does it always work?

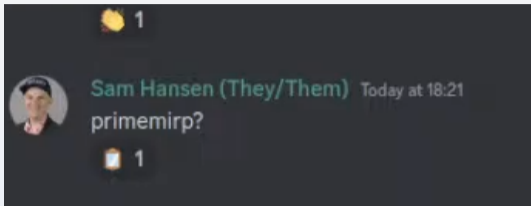
$$19 \rightarrow 91$$

Does it always work?

$$19 \rightarrow 91 = 7 \times 13$$

► No!

► In the comments...



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Overlapping the middle digit. Why?

- ▶ For prime $a_1 a_2 \dots a_n$ and emirp $a_n \dots a_2 a_1$.

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- ▶ Each a_i is in both an even place and an odd place.
- ▶ Since

$$10^k \pmod{11} = \begin{cases} 10, & \text{if } k \text{ is even;} \\ 1, & \text{otherwise,} \end{cases}$$

each a_i contributes a multiple of eleven (i.e. not prime).

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each a_i contributes a multiple of eleven (i.e. not prime).

- ▶ An overlapping middle digit ('primemirp' rather than 'primeemirp') avoids this by making the pair of a_i either odd or even but not both.

primemirp

- ▶ Does it work?

13

907

9103

primemirp

- ▶ Does it work?

13 & 31
907 & 709
9103 & 3019

primemirp

► Does it work?

13 & 31 → 131
907 & 709 → 90709
9103 & 3019 → 9103019

primemirp

- ▶ Does it work?

$$\begin{array}{rclcl} \underline{13} & \& 31 & \rightarrow & \underline{131} \\ \underline{907} & \& 709 & \rightarrow & \underline{90709} \\ \underline{9103} & \& 3019 & \rightarrow & \underline{9103019} \end{array}$$

- ▶ Yes!

primemirp

- ▶ Does it always work?

17

primemirp

- ▶ Does it always work?

17 & 71

primemirp

- ▶ Does it always work?

$$\underline{17} \ \& \ 71 \ \rightarrow \ \underline{171}$$

primemirp

- ▶ Does it always work?

$$\underline{17} \ \& \ 71 \ \rightarrow \ \underline{171} = 9 \times 19$$

- ▶ No!

**CAN HUMANS SAY THE LARGEST PRIME
NUMBER BEFORE WE FIND THE NEXT ONE?**



saythepime.com

- ▶ I wrote some code which:
 - found primes;
 - reversed them, checked whether the reversal was prime;
 - concatenated the primes and emirps and checked if they were prime.

```
for i in range(2,10**7):  
    if isprime(i):  
        emirp = int(str(i)[ 1:: 1])  
        if isprime(emirp):  
            primemirp = int(str(i)+str(emirp)[1:])  
            if isprime(primemirp):  
                print(primemirp)
```

- ▶ I found a few!

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0 1 3 6 2 7
: 13
: OE 20
23 IS 12
10 22 11 21

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founded in 1964 by N. J. A. Sloane

The On-Line Encyclopedia of Integer Sequences (OEIS)

Enter a sequence, word, or sequence number:

2,3,5,7,131,313,373,797,11311,17971,18181,19991,35353,72727,78787,90709

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(Greetings from [The On-Line Encyclopedia of Integer Sequences!](#))

Search:

seq:2,3,5,7,131,313,373,797,11311,17971,18181,19991,35353

Displaying 1-1 of 1 result found. page 1

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[A054218](#) Palindromic primes of the form 'primemirp' +30
3
resulting from [A054217](#).

**2, 3, 5, 7, 131, 313, 373, 797, 11311, 17971, 18181,
19991, 35353, 72727, 78787, 90709, 93739, 96769,
98389, 1153511, 1193911, 1201021, 1409041, 1583851, 1597951,**

primemirp

- ▶ Added to OEIS in February 2000 by Patrick De Geest.
- ▶ Based on an idea from G. L. Honaker, Jr.

primemirp

- ▶ Added to OEIS in February 2000 by Patrick De Geest.
- ▶ Based on an idea from G. L. Honaker, Jr.
- ▶ But there was no code, and only 32 examples were given.
- ▶ I edited the entry to add a Python program and entries up to the 8,668th, which I believe is all primemirps where the underlying prime is less than ten million.
- ▶ The 8,668th primemirp is 9,999,713,179,999.

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0 1 3 6 2 7
: 13
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10 22 11 21

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A054218 Palindromic primes of the form 'primemirp' resulting from [A054217](#).³

2, 3, 5, 7, 131, 313, 373, 797, 11311, 17971, 18181, 19991, 35353, 72727, 78787, 90709, 93739, 96769, 98389, 1153511, 1193911, 1201021, 1409041, 1583851, 1597951, 1657561, 1831381, 1879781, 3083803, 3089803, 3319133, 3343433, 3391933, 3541453, 3643463

(list; [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET 1,1

COMMENTS Original idea from [G. L. Honaker, Jr.](#)

LINKS Peter Rowlett, [Table of n, a\(n\) for n = 1..8668](#)

EXAMPLE Prime 113 has emirp 311 and 11311 is a palindromic prime.

PROG (Python)
from sympy import isprime
for i in range(2, 10**7):
 if isprime(i):



Thanks for
listening!



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